A Wavefront-Like Strategy for Computing Multiplicatively Weighted Voronoi Diagrams

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March 19, 2019

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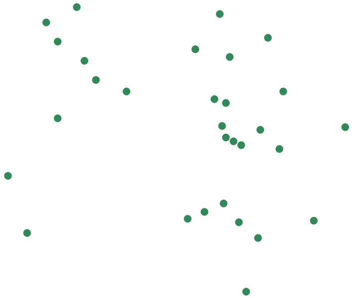
Problem Specification

Given:

A set S of n input points in the plane, where every $s \in S$ is associated with a real-valued weight $w(s) > 0$.



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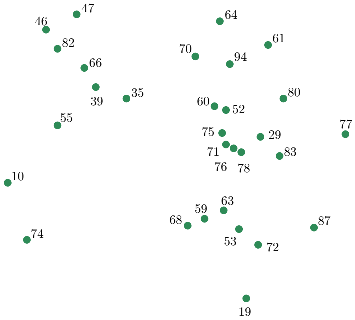

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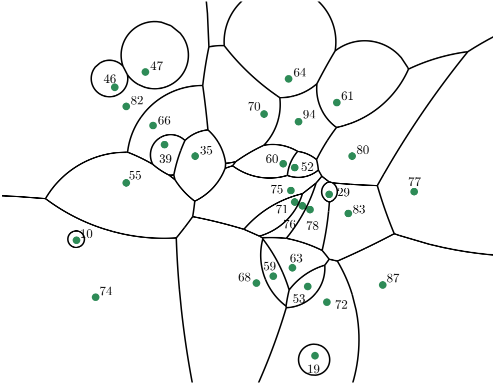
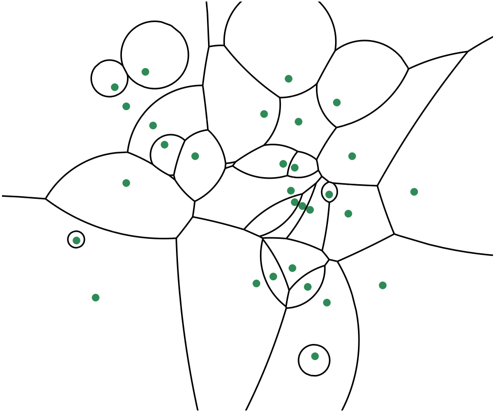
Compute:

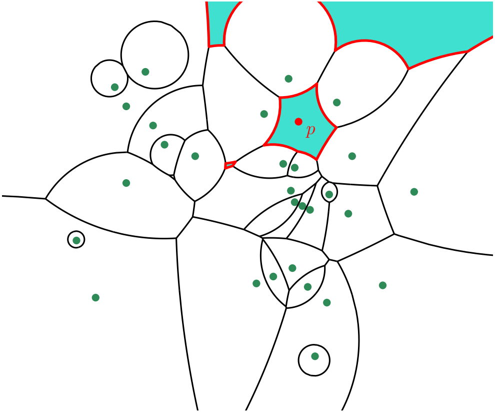
The multiplicatively weighted Voronoi diagram (MWVD) $\mathcal{VD}_w(S)$ of S.



- The Voronoi edges are formed by straight-line segments and circular arcs.

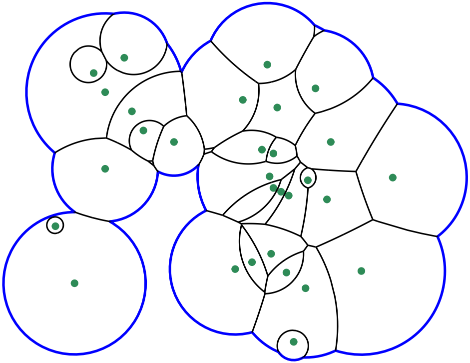


- The Voronoi edges are formed by straight-line segments and circular arcs.
- The Voronoi regions are (possibly) multiply connected.
- The MWVD has a quadratic combinatorial complexity in the worst case.



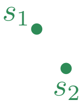
Overview

- We present a wavefront-based approach for computing MWVDs.
- The ***wavefront*** covers an increasing portion of the plane over time.
- It consists of ***wavefront arcs*** and ***wavefront vertices***.
- Whenever a wavefront arc vanishes or spawns, a new Voronoi node is discovered.



Offset Circles

- Every site is associated with an ***offset circle***.



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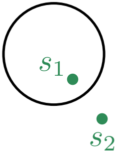

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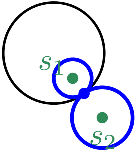
- Every site is associated with an ***offset circle***.
- Two ***moving intersection points*** trace out the bisector as time progresses.



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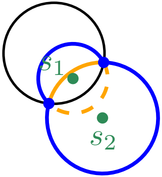
- Every site is associated with an ***offset circle***.
- Two ***moving intersection points*** trace out the bisector as time progresses.
- ***Inactive arcs*** along the offset circles are eliminated.
- The ***active arcs*** are stored in sorted angular order.



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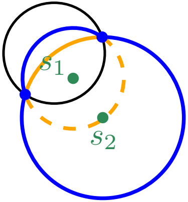

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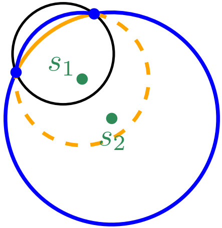

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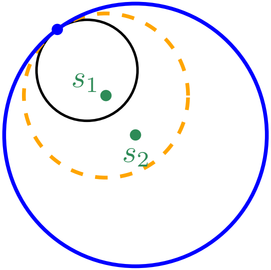
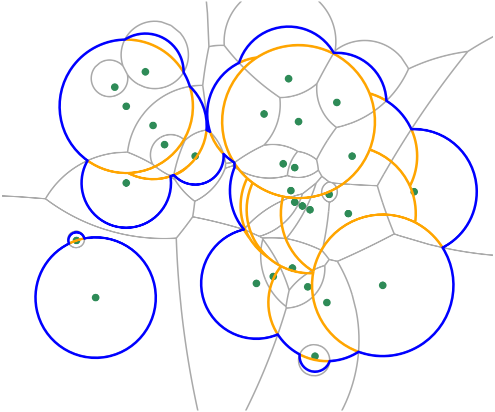


Wavefront Structure

- The wavefront is formed by parts of offset circles.
- A wavefront vertex is a specific intersection point of two offset circles.
- Active arcs do **<u>not</u>** necessarily coincide with wavefront arcs.



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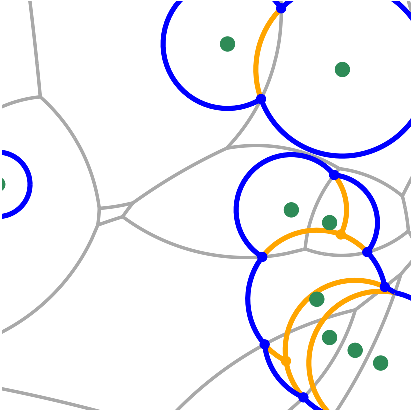
- ***Collision*** and ***domination events*** mark the initial and last contact (of a pair of offset circles), respectively.
- ***Edge*** and ***break-through events*** happen whenever active arcs vanish or spawn.
- These events are stored in a priority queue $\mathcal{Q}$.



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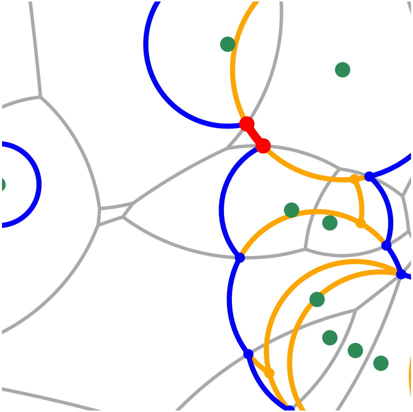

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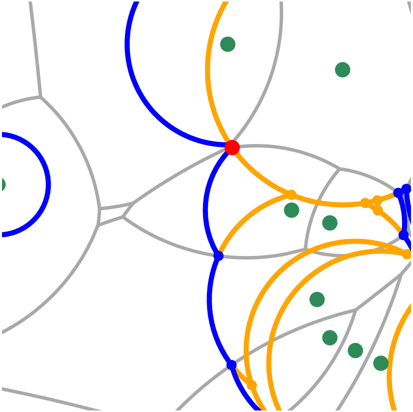

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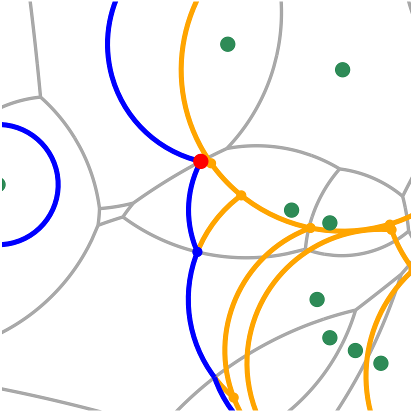

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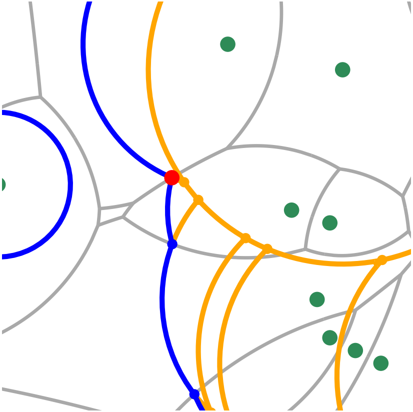

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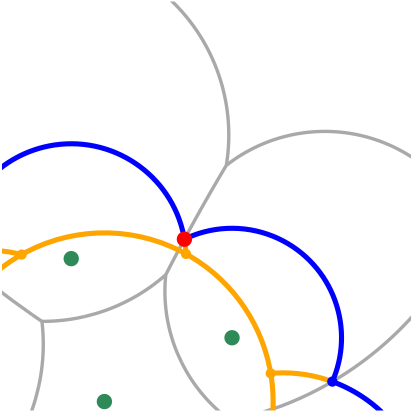

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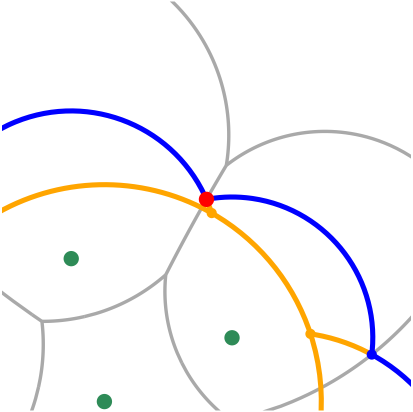

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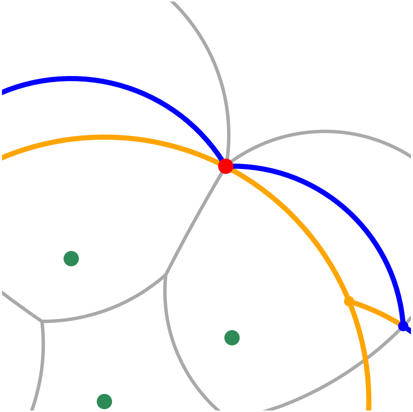

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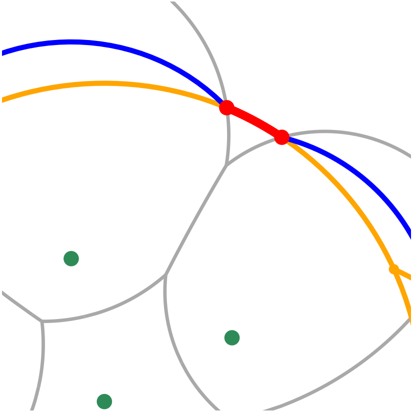

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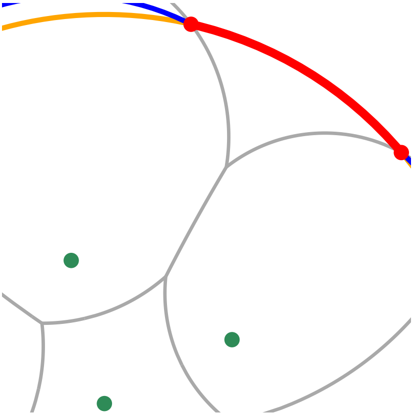

Event Handling

- Initially, the collisions of every pair of offset circles are computed.
- Afterwards, the events are successively popped from $\mathcal{Q}$.
- A collision event is ***valid*** if the collision point is situated within active arcs.

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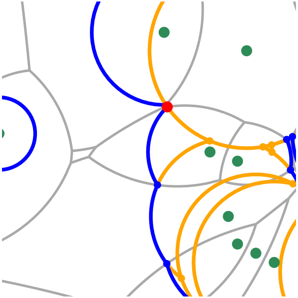
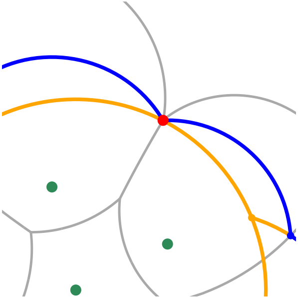
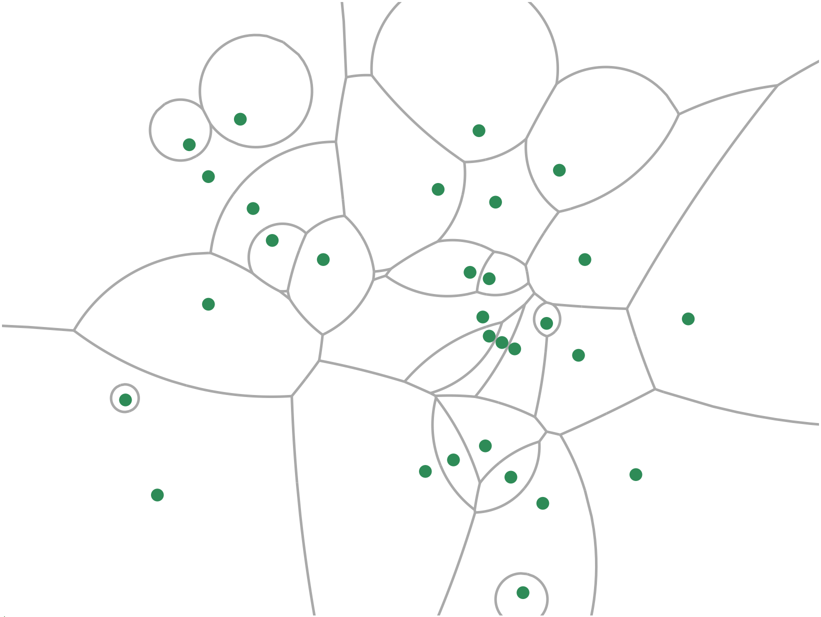
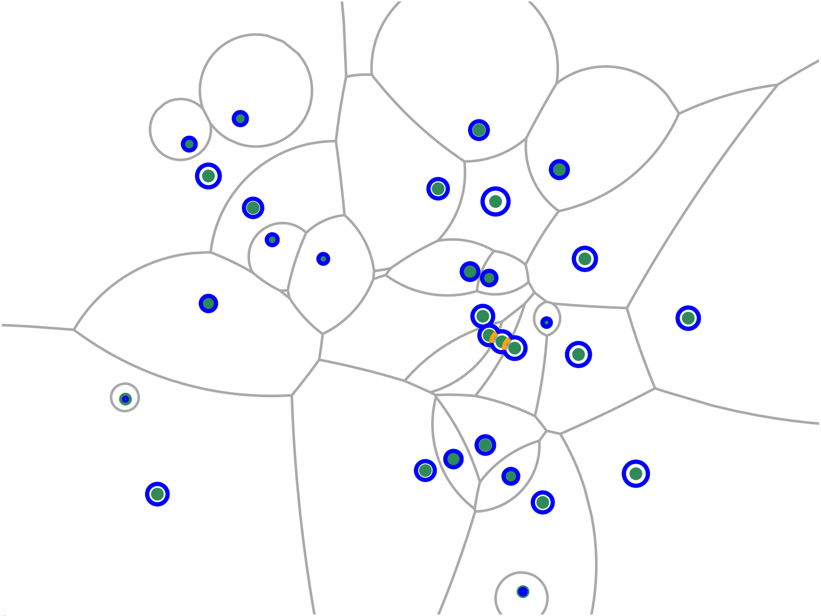
- A collision event is ***valid*** if the collision point is situated within active arcs.
- On each edge and break-through event, three offset circles need to be updated.
- The angular order of active arcs only changes at events.







Wavefront Propagation



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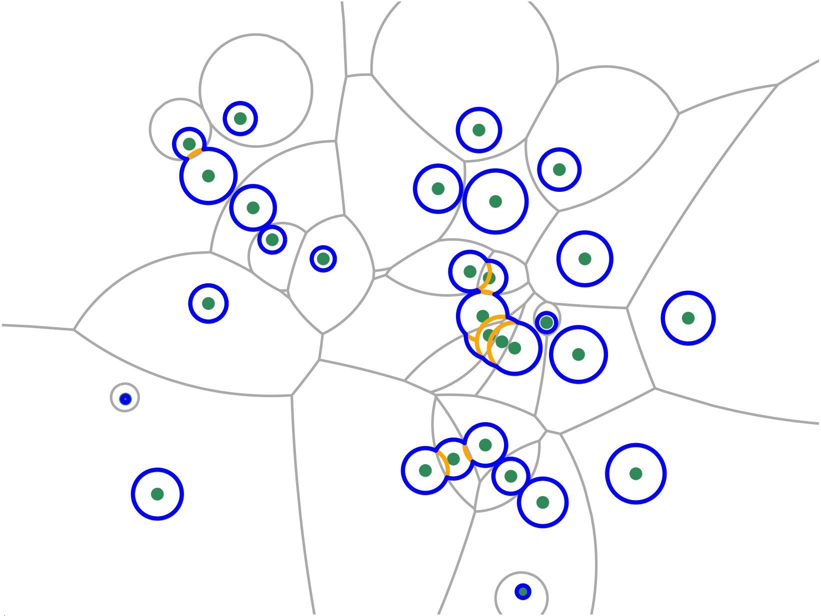

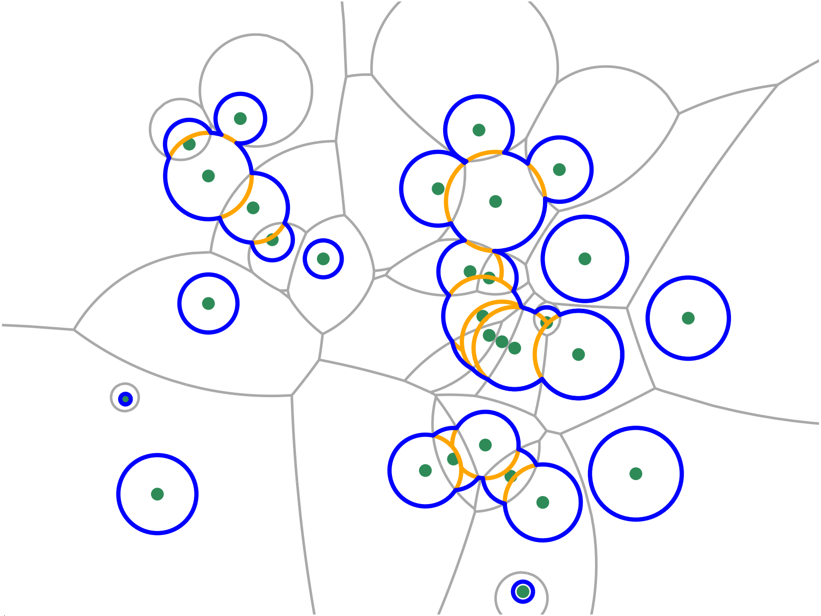

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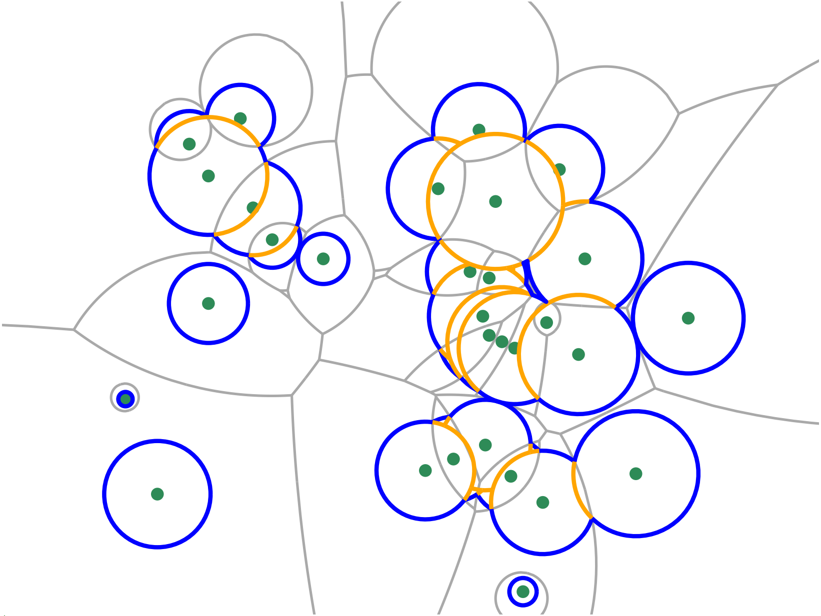

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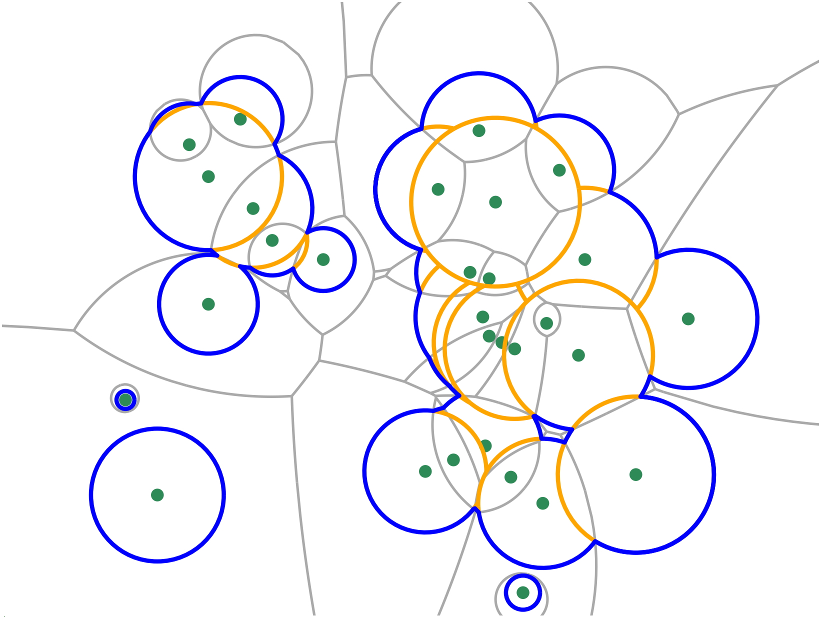

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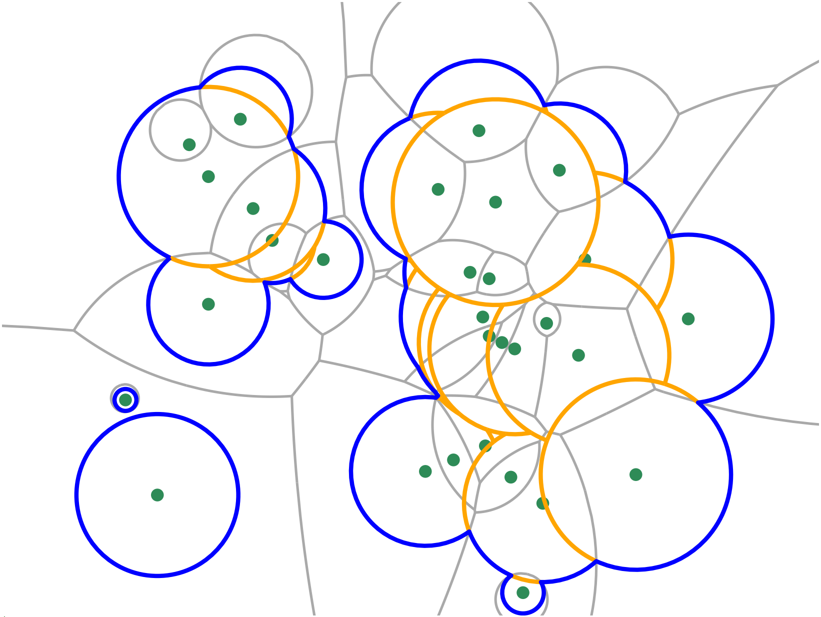

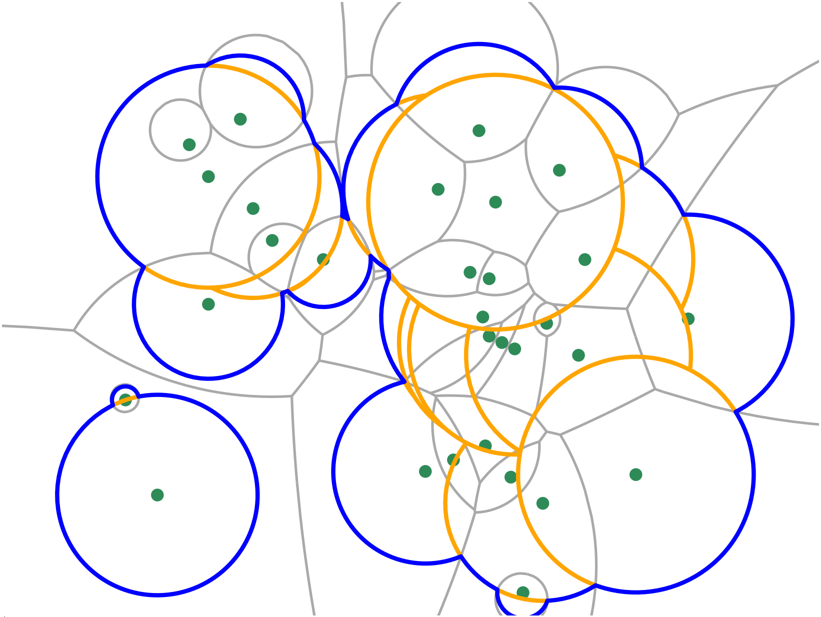

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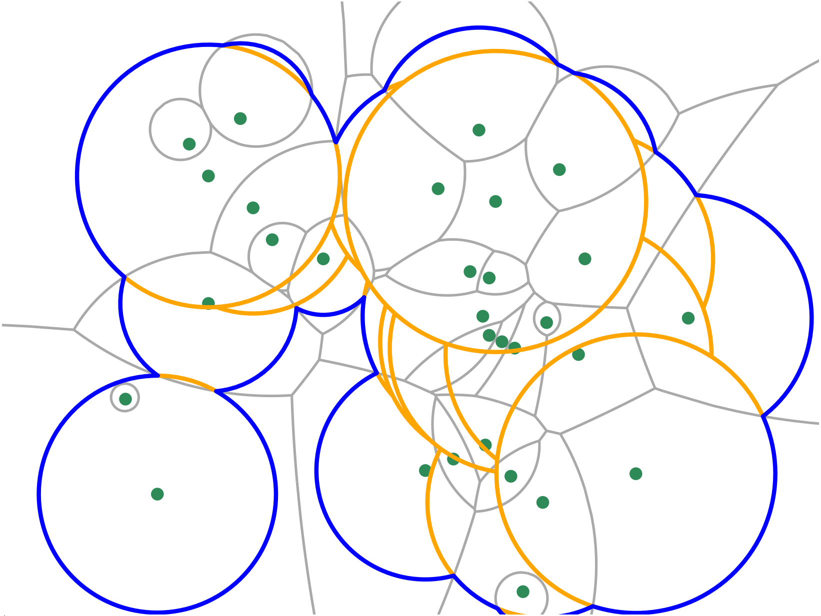

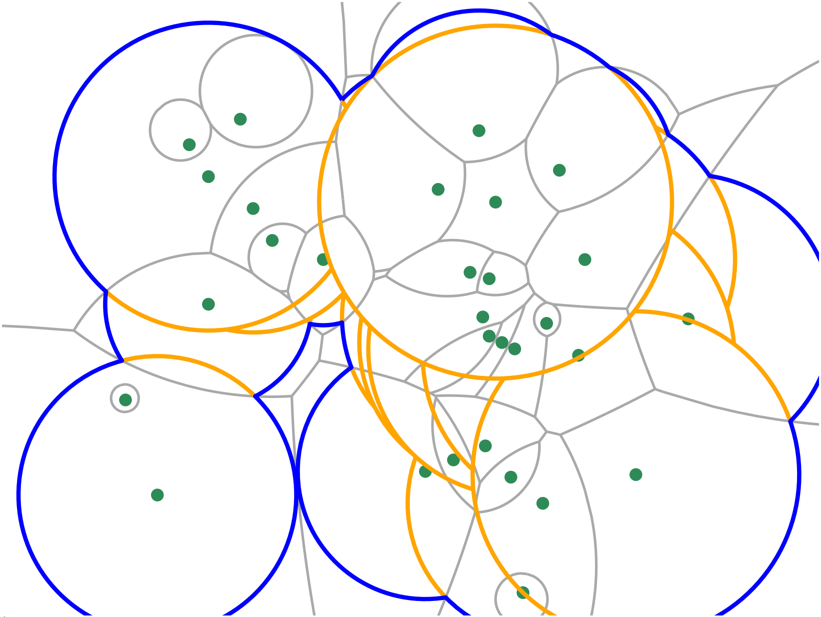

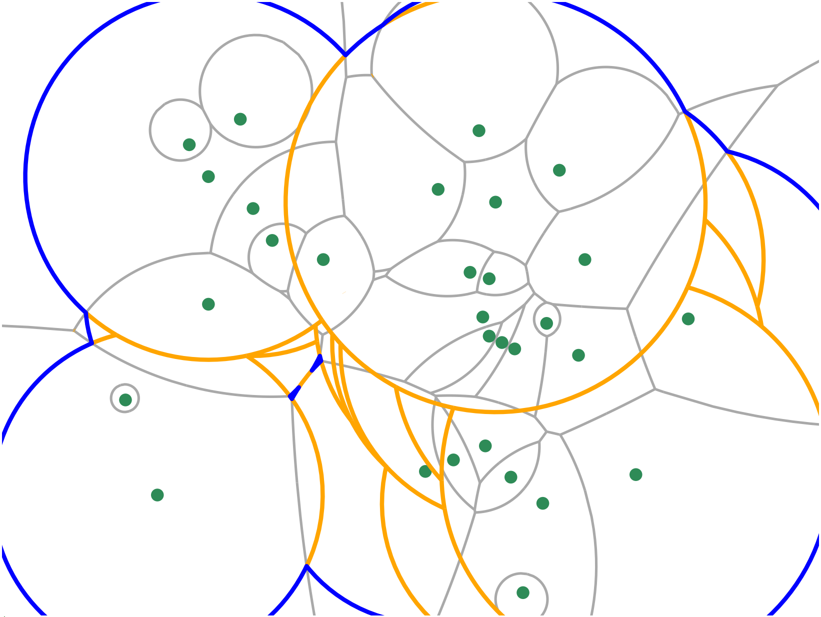

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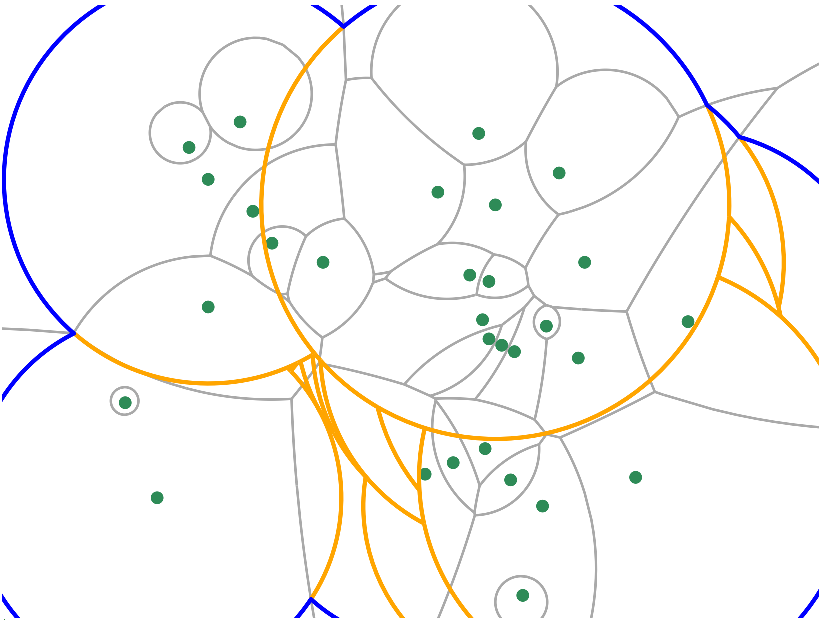
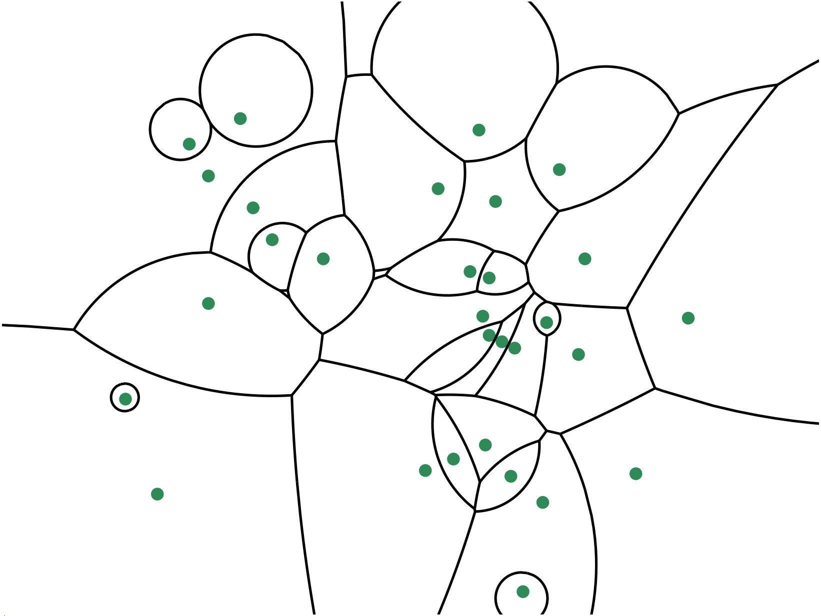
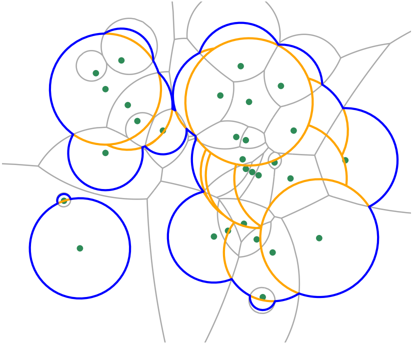




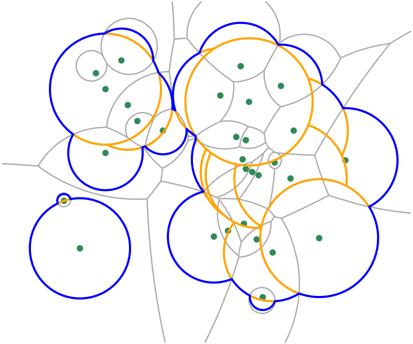
- All topological changes of the wavefront are properly detected.
- A quadratic number of collision events are computed in any case.
- In the worst case $\mathcal{O}(n^2)$ break-through events take place.
- At most $\mathcal{O}(n^2)$ active arcs are generated during the wavefront propagation.
- Therefore, the algorithms runtime is $\mathcal{O}(n^2 \log n)$ in the worst case.

- Our algorithm is also able to handle additive weights.



Discussion

- Our algorithm is also able to handle additive weights.
- Improve the average case behavior of our strategy.
- Reduce the number of collisions that are computed.



Thank you for your attention!