Computing Low-Cost Convex Partitions for Planar Point Sets Based on Tailored Decompositions

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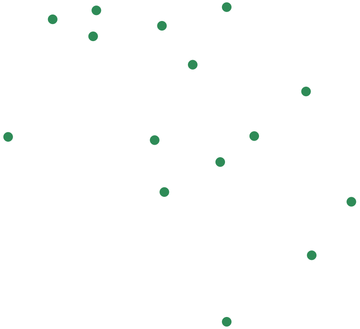
Computational Geometry and Applications Lab

Problem Specification

Problem

Given: A set P of n points in the plane.



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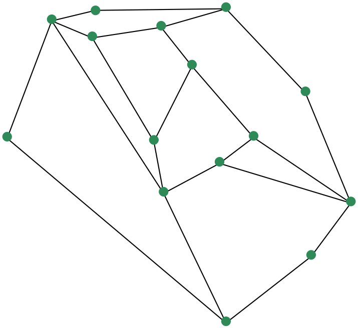
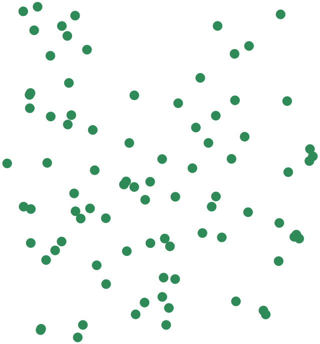
Given: A set P of n points in the plane.
Compute: A plane graph with vertex set P (with each point in P having positive degree) that partitions the convex hull of P into the smallest possible number of convex faces. Note that collinear points are allowed on face boundaries, so all internal angles of a face are at most π.



3-Approximation

- The 3APX tool implements the algorithm by Knauer and Spillner [KS06].



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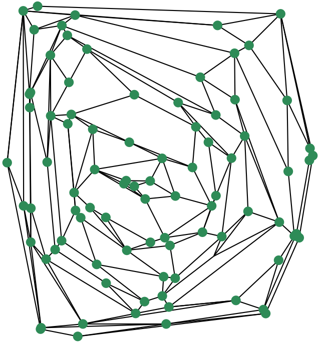
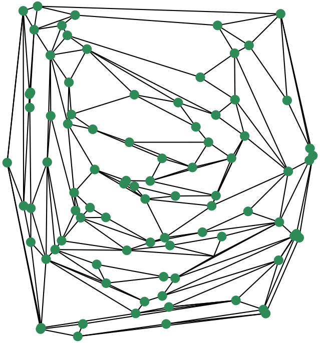


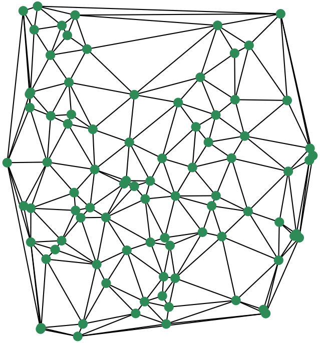
3-Approximation

- The 3APX tool implements the algorithm by Knauer and Spillner [KS06].
- We extended 3APX by an approach based on onion layers.
- The decompositions generated contained lots of extremely long and thin triangles.
- A partitioning into smaller “cells” did not reduce the face counts substantially.



Merging Triangles

- **Simple idea:** Start with a Delaunay triangulation ...



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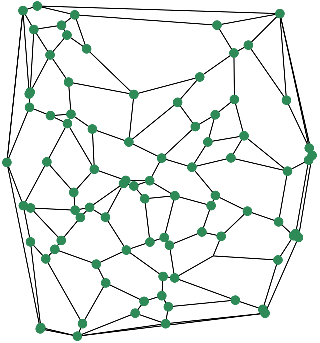

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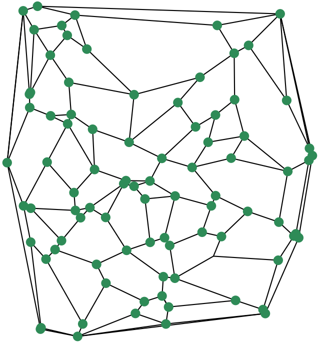
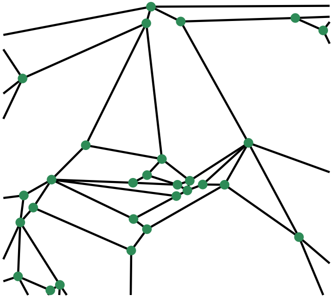
- Our first implementation MergeRefine easily beat 3Apx.
- This initial success motivated the development of a more sophisticated strategy.



An Improved Implementation

- RECURSOR introduces several heuristics.



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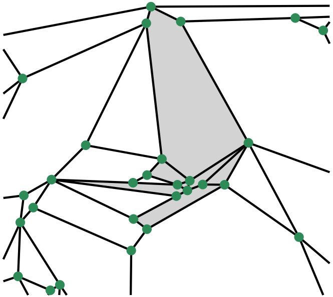

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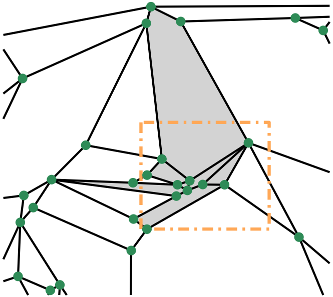

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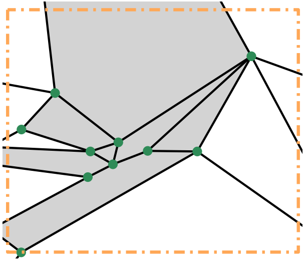

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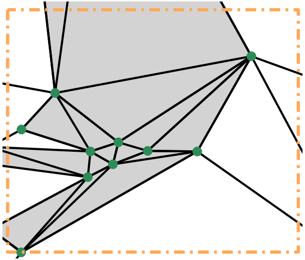

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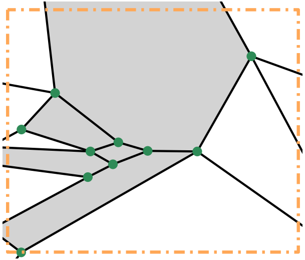

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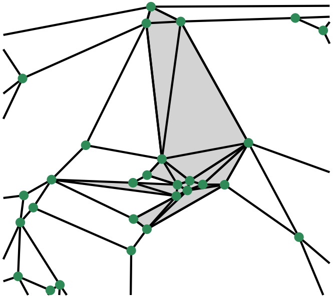

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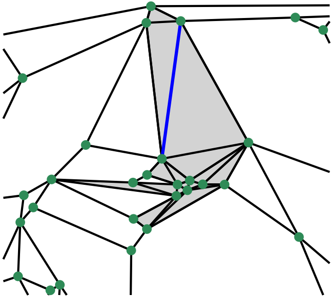

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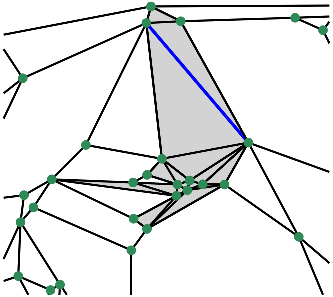

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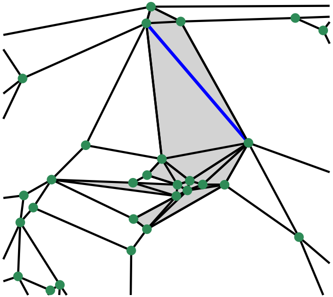

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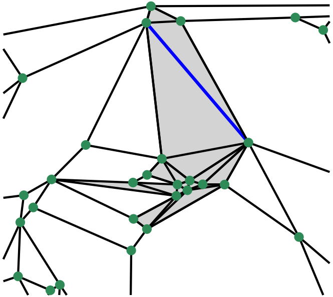
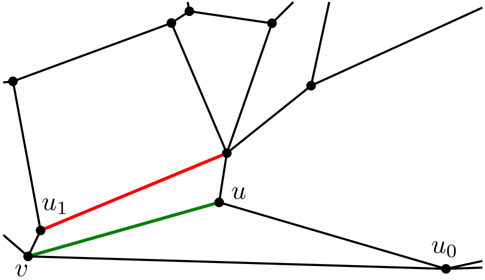
- **Parallel recursor**: Partition a decomposition into several non-overlapping sets of faces.



Flipping Edges

- FLIPPER was implemented relatively late.
- It picks a high degree vertex and rotates incident edges.
- Unnecessary edges are removed.
- FLIPPER interacts with RECURSOR as it re-structures the respective decompositions.



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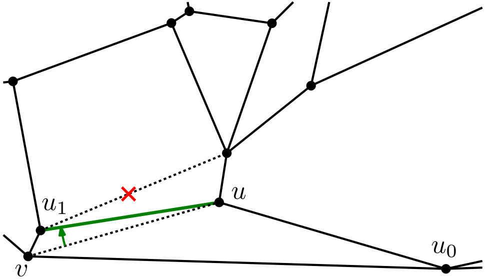
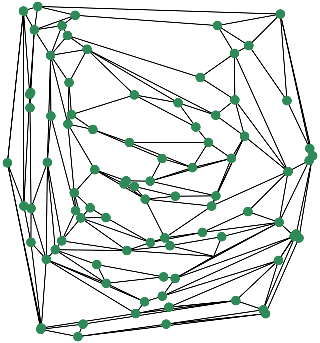


Example Decomposition

3APX (random)

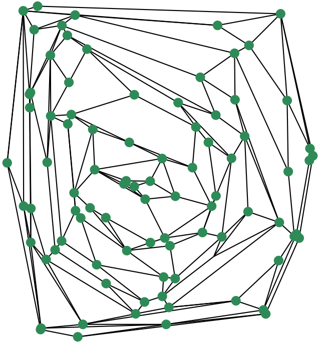
#Faces 111



Example Decomposition

3APX (random) → 3APX (onion)

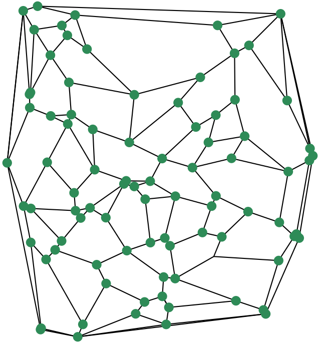
#Faces 100



Example Decomposition

3APX (random) → 3APX (onion) → MERGEREFINE

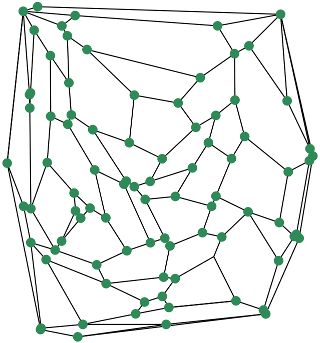
#Faces 63



Example Decomposition

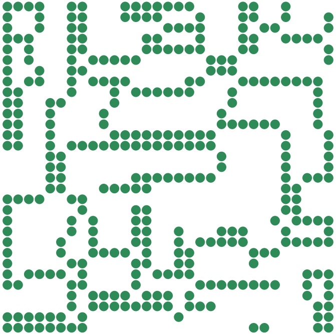
3APX (random) → 3APX (onion) → MERGEREFINE → RECURSOR + FLIPPER

#Faces 54



Orthogonal Optimizer

- Later on a second batch of instances was made available.
- These instances contained points sampled from a dense integer grid.



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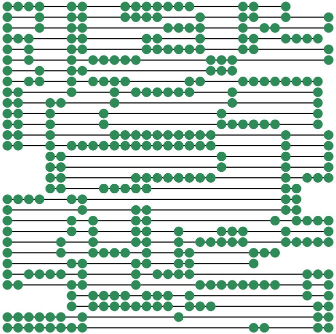
- ORTHOOPT generates initial decompositions that are geared towards this type of input sets.



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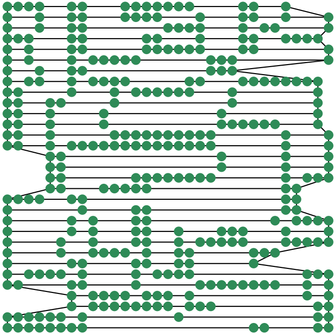

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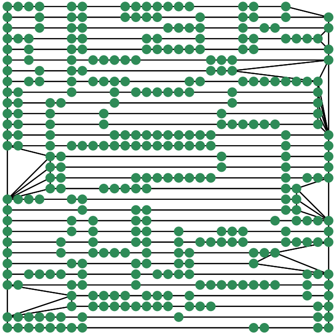

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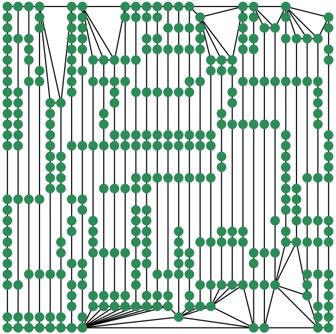

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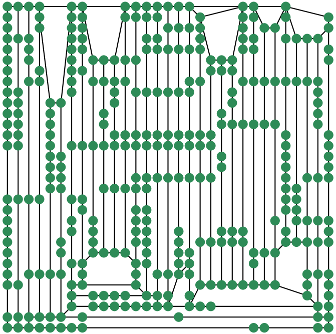
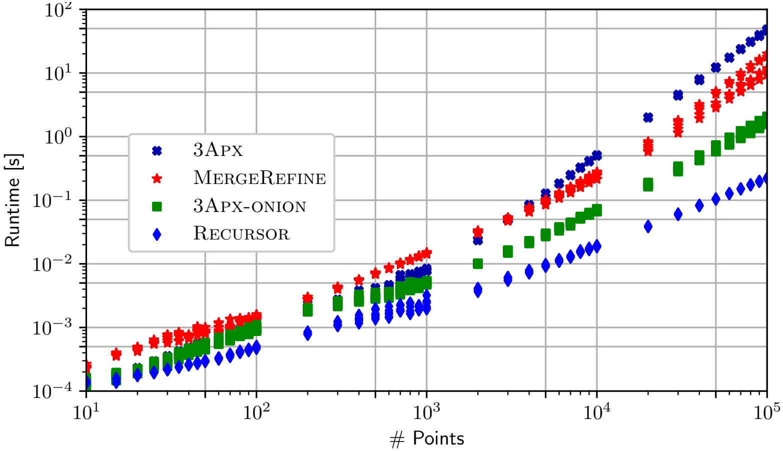
- These initial decompositions were improved by Flipper and Recursor.



Experimental Results

- All runtime tests were carried out on an Intel Core i7-6700 CPU clocked at 3.40 GHz.



Average Score Over Time

- We ran our tools on a wide variety of different computers.

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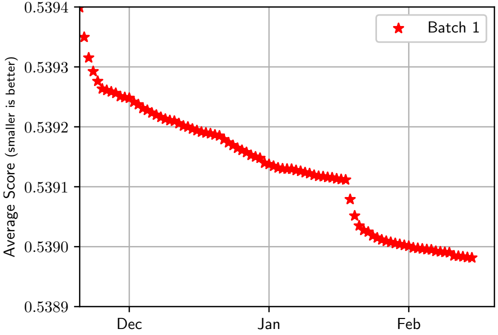
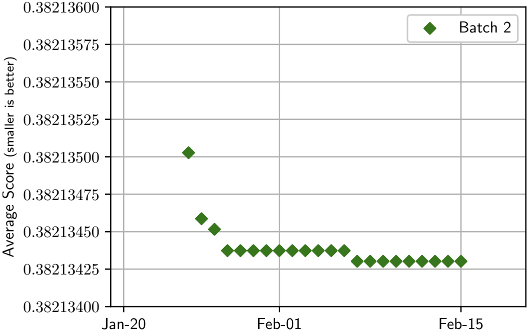
- The estimated quality of a given decomposition is based on its ***score***.

$$\text{score} := \frac{\text{number of edges in convex partition}}{\text{number of edges in triangulation}}$$





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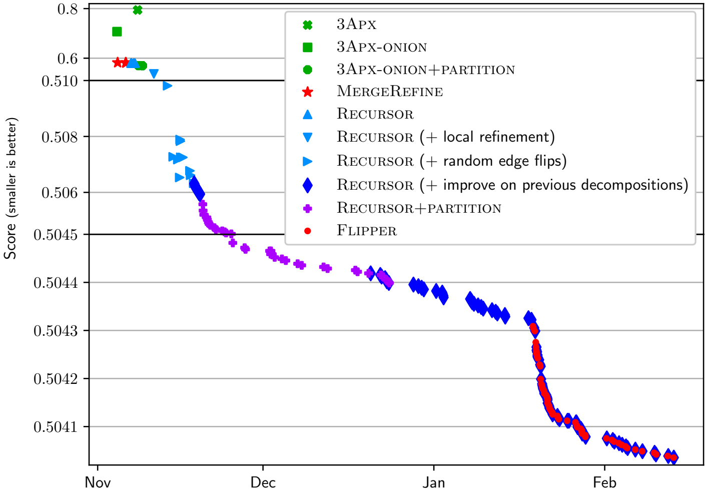
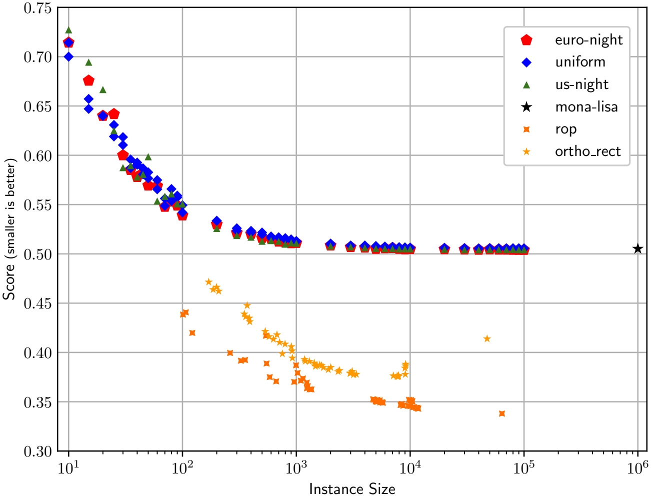


Score Per Instance



Thank You For Your Attention!

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References I

Christian Knauer and Andreas Spillner.
Approximation Algorithms for the Minimum Convex Partition Problem.
In *Algorithm Theory – SWAT 2006*, pages 232–241, 2006.