Generalized Voronoi Diagrams: Theory and Related Applications

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October 5, 2021

UNIVERSITÄT SALZBURG
Computational Geometry and Applications Lab

Overview

An Efficient, Practical Algorithm and Implementation for Computing Multiplicatively Weighted Voronoi Diagrams
HELD AND DE LORENZO
Published in *Proceedings of the 28th Annual European Symposium on Algorithms (ESA 2020)*

Weighted Skeletal Structures for Computing Variable-Radius Offsets
HELD AND DE LORENZO
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On the Recognition and Reconstruction of Weighted Voronoi Diagrams and Bisector Graphs
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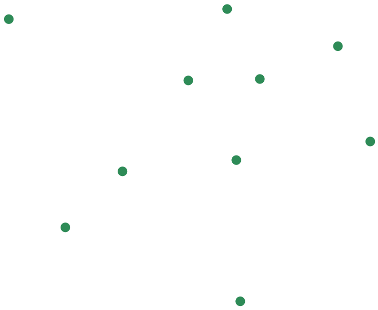
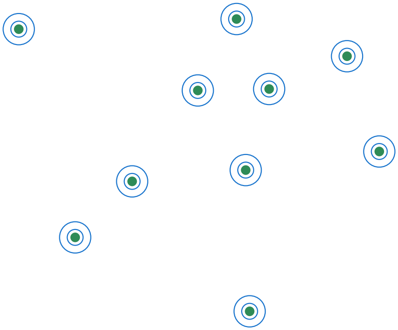
Wavefront Propagation





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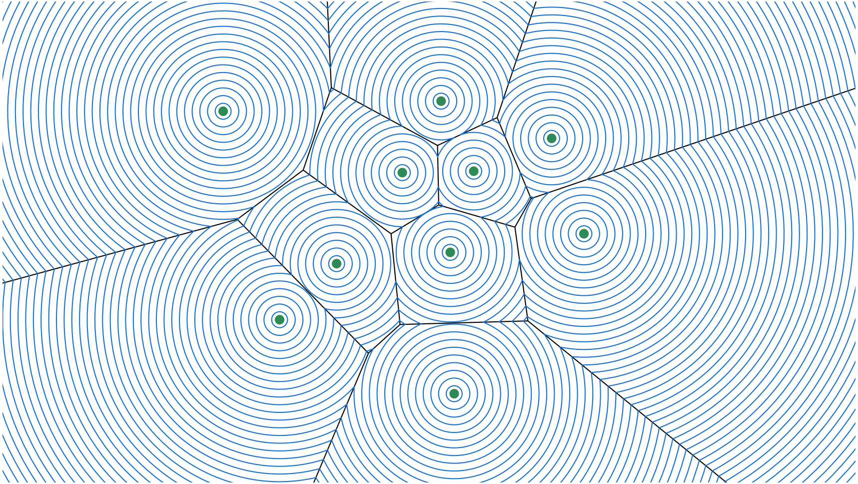

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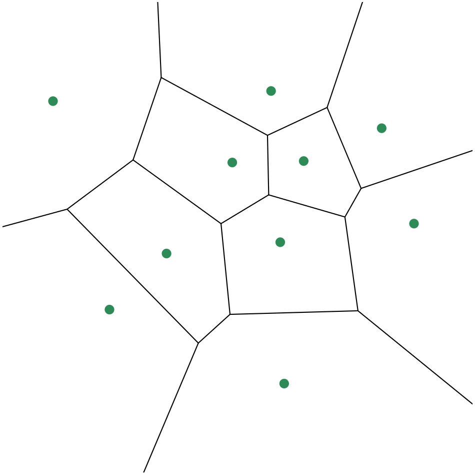
The Voronoi Diagram of Points



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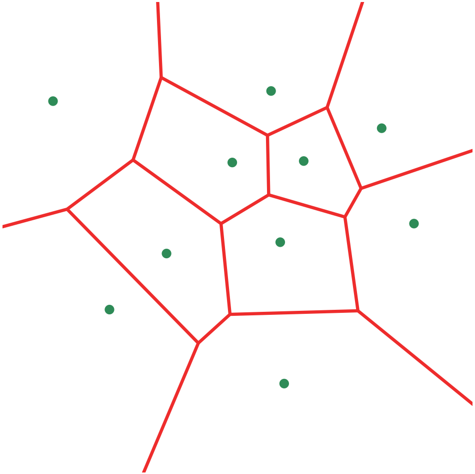

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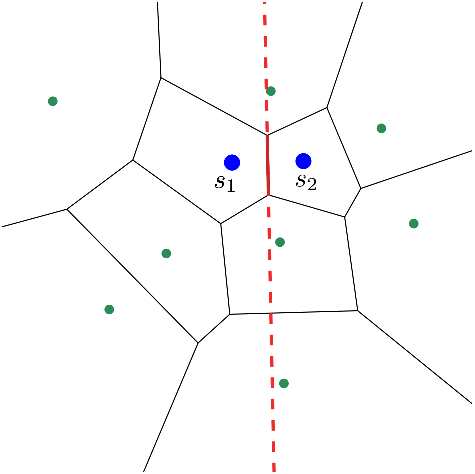

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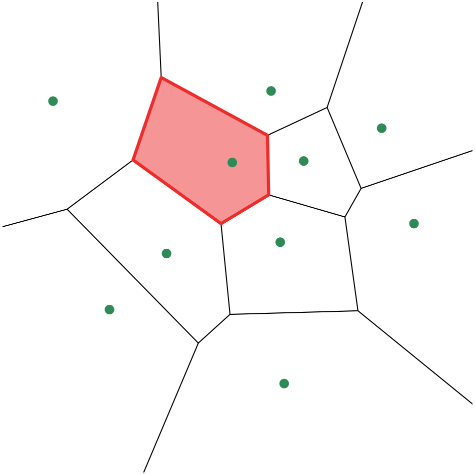

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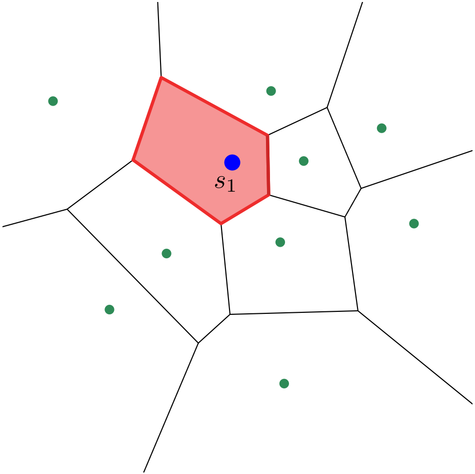

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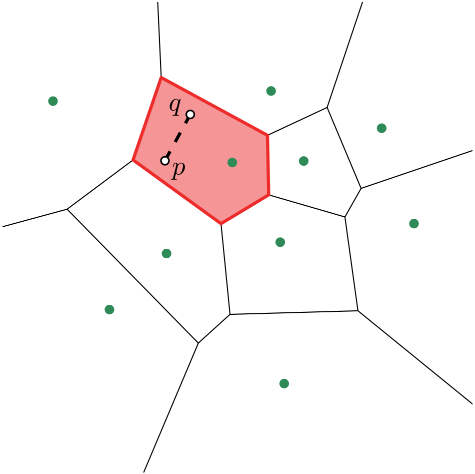

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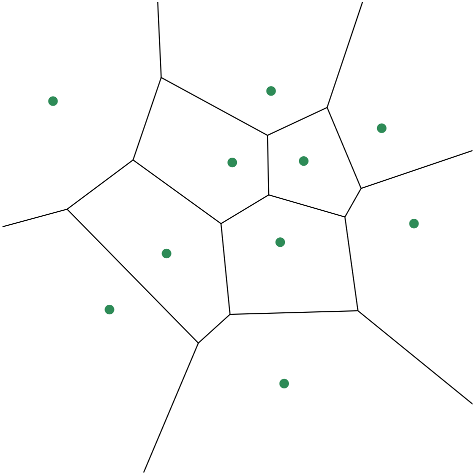
- Each Voronoi region is ***convex***.

The Delaunay Triangulation



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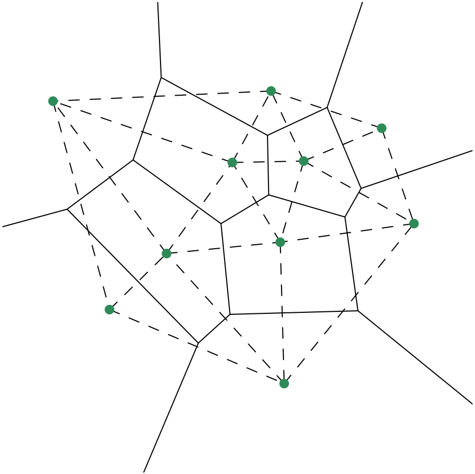

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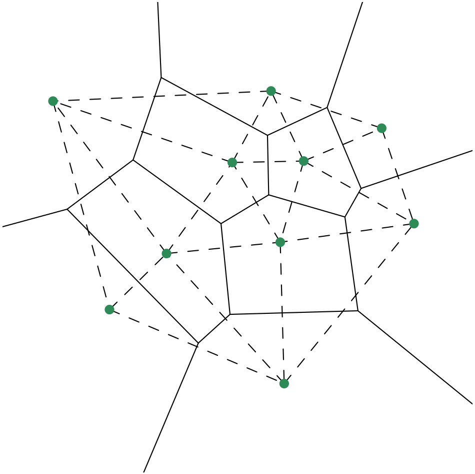

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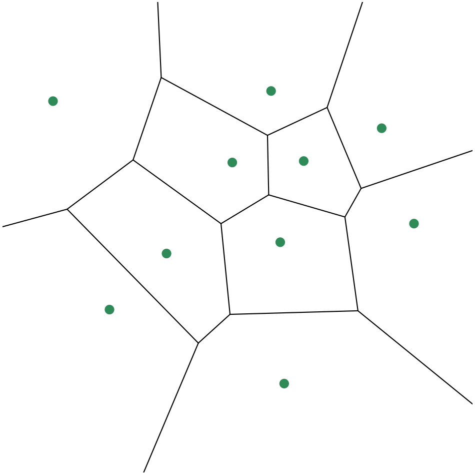
- It maximizes the minimum angle inside a triangle over all possible triangulations.



Theorem (Shamos and Hoey 1975)

The Voronoi diagram of n input points can be computed in $\mathcal{O}(n \log n)$ time and $\mathcal{O}(n)$ space using a divide-and-conquer strategy.

Theorem (Fortune 1987)

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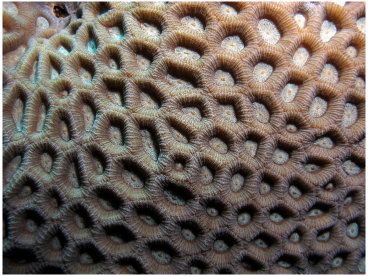
Voronoi Diagrams in Nature



Credit: *Keats 2009*

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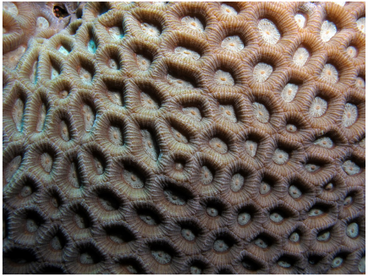

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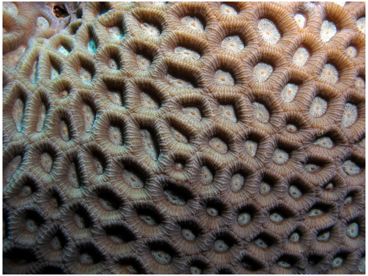

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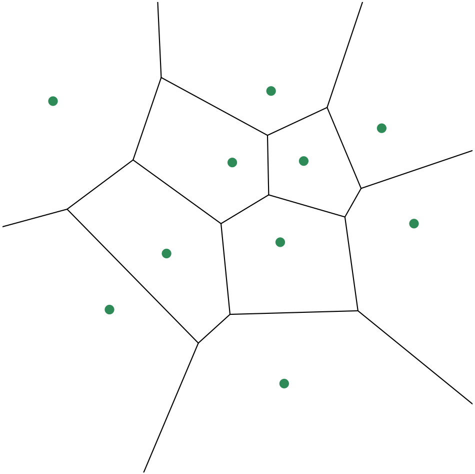

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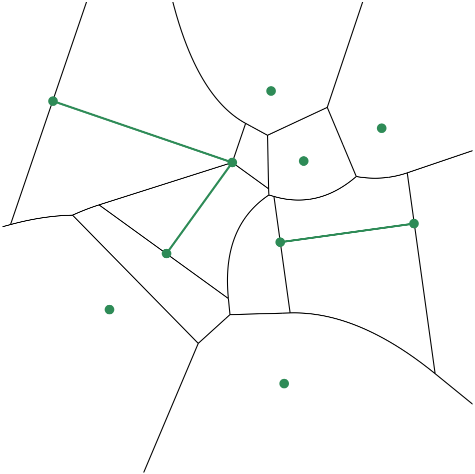

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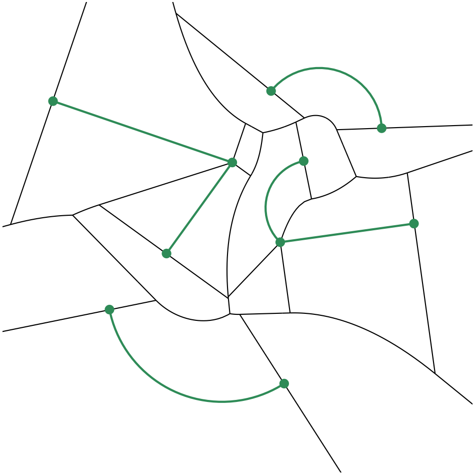

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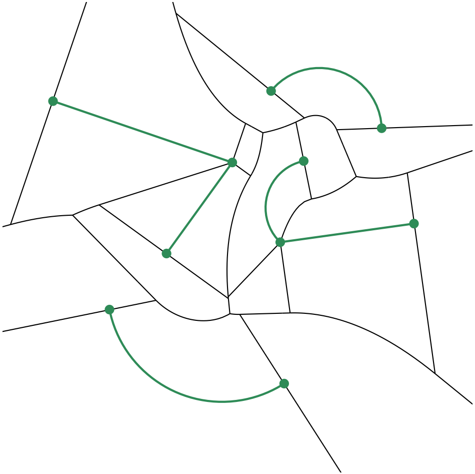
Allowing Other Types of Input Sites



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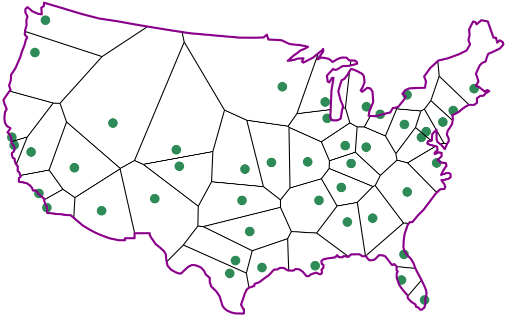
Theorem (Yap 1987)

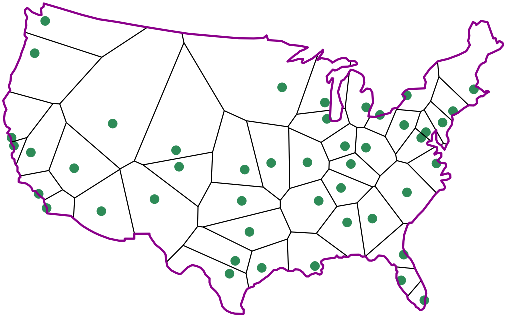
It is possible to generate the Voronoi diagram of n straight-line segments and circular arcs in optimal $\mathcal{O}(n \log n)$ time.

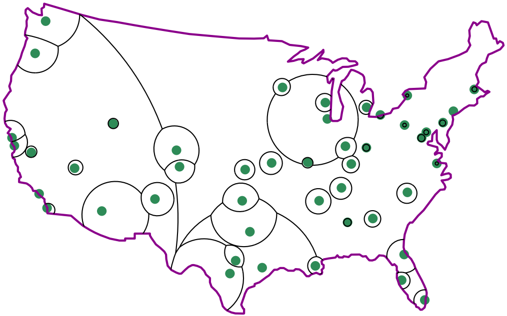
Theorem (Held and Huber 2009)

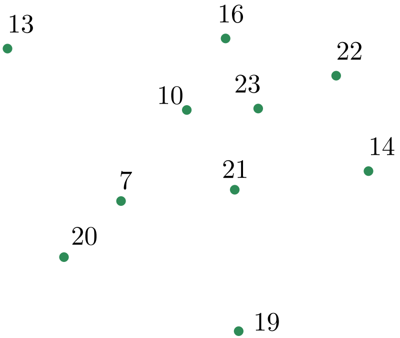
The Voronoi diagram of n points, straight-line segments, and circular arcs can be computed in expected $\mathcal{O}(n \log n)$ time.

Multiplicatively Weighted Voronoi Diagrams

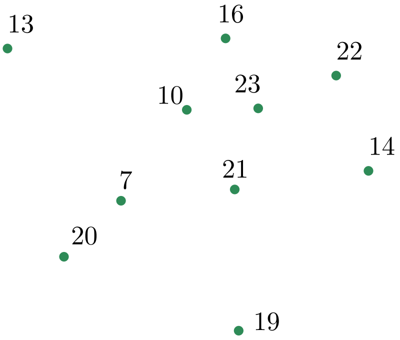








- Consider a set S of point sites in the Euclidean plane.
- We associate every site s with a real-valued ***weight*** $w(s) > 0$.



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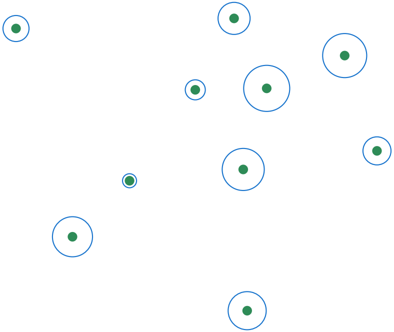
The ***(multiplicatively) weighted distance*** $d_w(p, s)$ between a weighted site $s \in S$ and a points $p \in \mathbb{R}^2$ is given by

$$d_w(p, s) := \frac{d(p, s)}{w(s)}.$$



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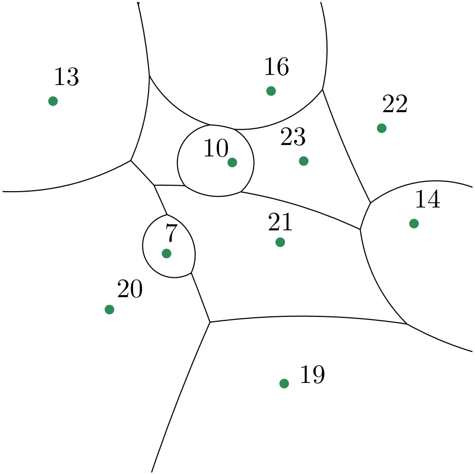

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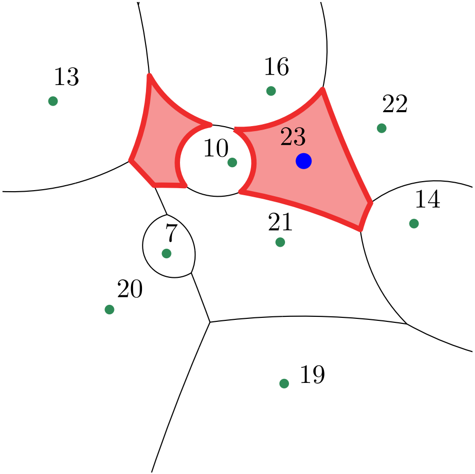

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- The Voronoi regions are (possibly) disconnected.

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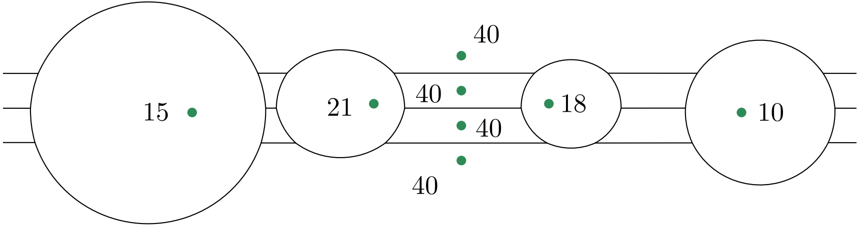
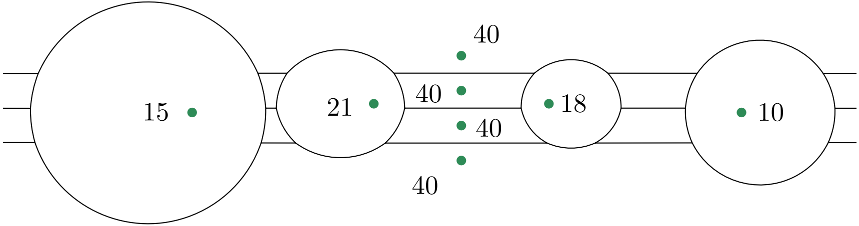


Theorem (Aurenhammer and Edelsbrunner 1984)

The multiplicatively weighted Voronoi diagram of n input sites has a combinatorial complexity of $\mathcal{O}(n^2)$ in the worst case.

A Worst-Case Example



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The multiplicatively weighted Voronoi diagram of n input sites can be computed in (optimal) $\mathcal{O}(n^2)$ time and space.

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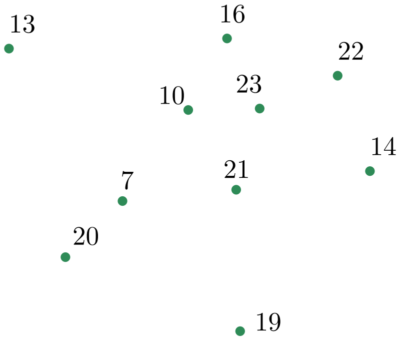
A Wavefront-Based Strategy



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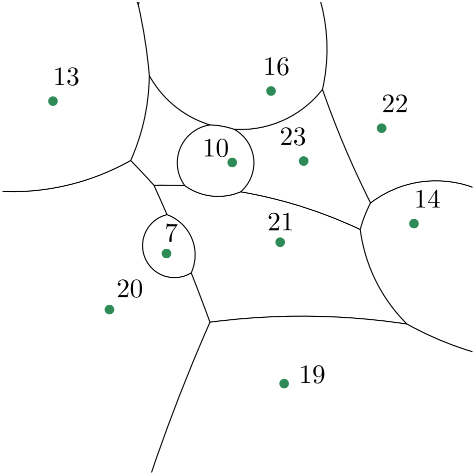

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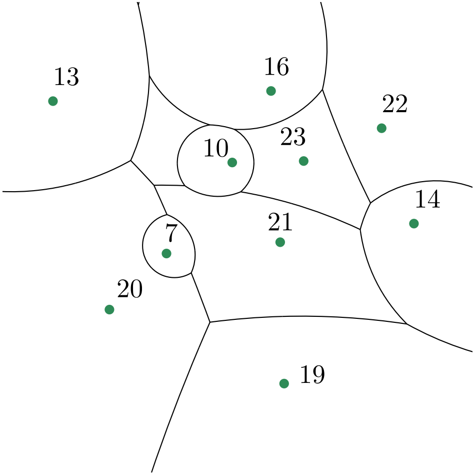

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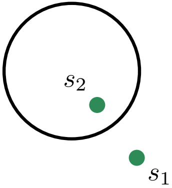
- We present a wavefront-based approach for computing the MWVD.
- The wavefront covers an increasing portion of the plane over time.
- It consists of wavefront arcs and wavefront vertices.
- Whenever a wavefront arc appears or disappears, a new Voronoi node is discovered.

Offset Circles



- Every site is associated with an ***offset circle***.

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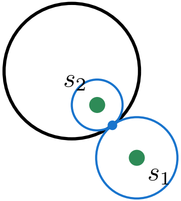

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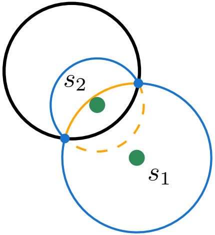

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- ***Inactive arcs*** along the offset circles are eliminated.
- The ***active arcs*** are stored in sorted angular order.

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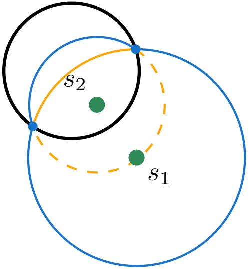

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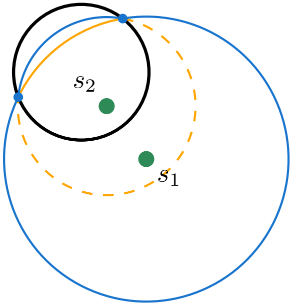

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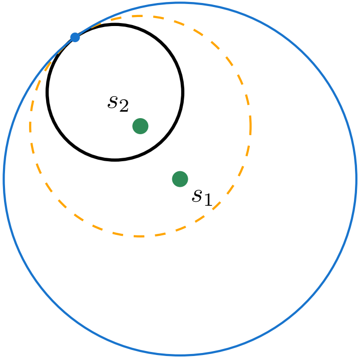

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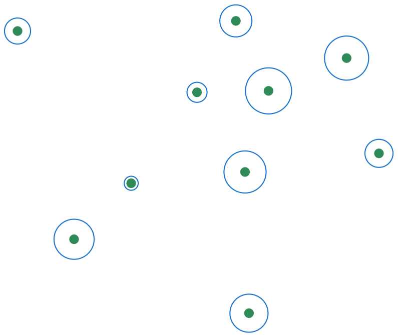

- ***Collision*** and ***domination events*** mark the initial and last contact of two offset circles.
- ***Arc events*** happen whenever active arcs appear or disappear.
- These events are stored in a priority queue.
- The angular order of active arcs only changes at events.

Event Handling

- All topological changes of the wavefront are properly detected.
- A quadratic number of collision events are computed in any case.
- A moving vertex can be charged with a constant number of arc events.
- In the worst case $\mathcal{O}(n^2)$ arc events take place.
- All events can be handled in $\mathcal{O}(\log n)$ time.

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Theorem (Held and de Lorenzo 2020)

The MWVD $\mathcal{VD}_w(S)$ of a set S of n weighted point sites in $\mathbb{R}^2$ can be computed in $\mathcal{O}(n^2 \log n)$ time and $\mathcal{O}(n^2)$ space.

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Theorem (Held and de Lorenzo 2020)

The MWVD $\mathcal{VD}_w(S)$ of a set S of n weighted point sites in one dimension can be computed in $\mathcal{O}(n \log n)$ time and $\mathcal{O}(n)$ space.

Theorem (Har-Peled and Raichel 2015)

Let S be a set of n points in the plane, where for each point we independently sample a weight from some distribution. Then the expected complexity of the MWVD of S is $\mathcal{O}(n \log^2 n)$.

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- A vast number of collisions are invalid for general input.
- The calculation of all possible collision requires a high computational effort.
- Invalid collision are filtered in an additional preprocessing step.

Reducing the Number of Collisions

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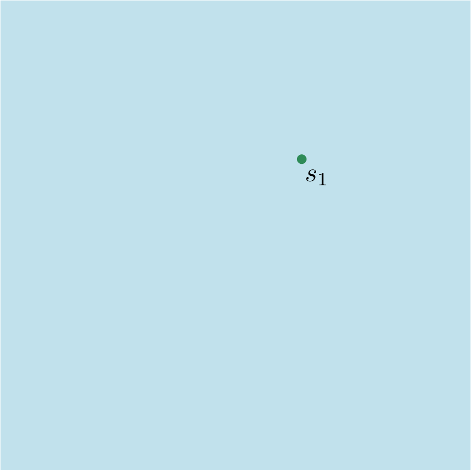
- Invalid collision are filtered in an additional preprocessing step.
- The average case behavior of the algorithm is improved by using an ***overlay arrangement***.

Reducing the Number of Collisions



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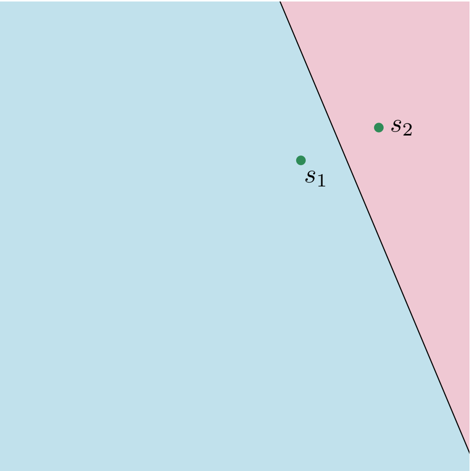

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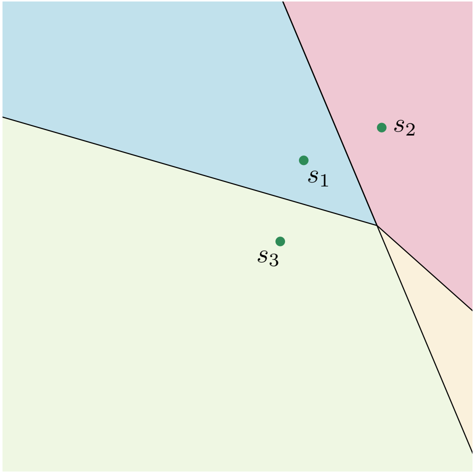

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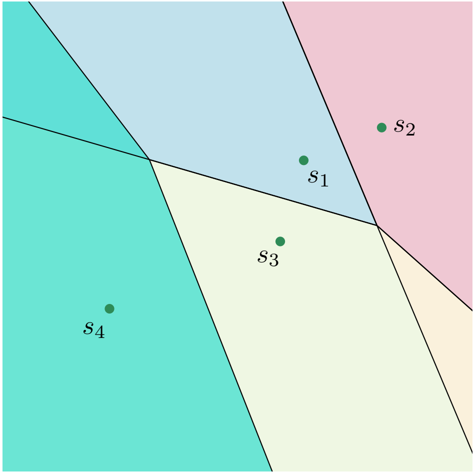

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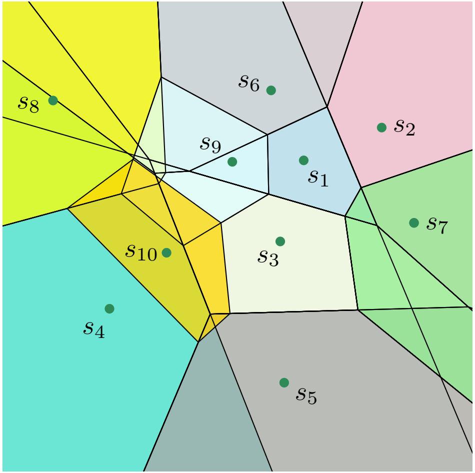

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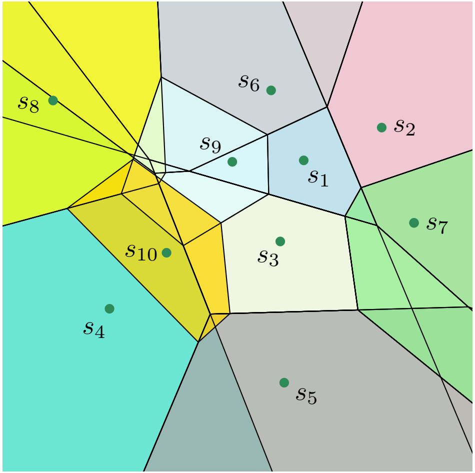
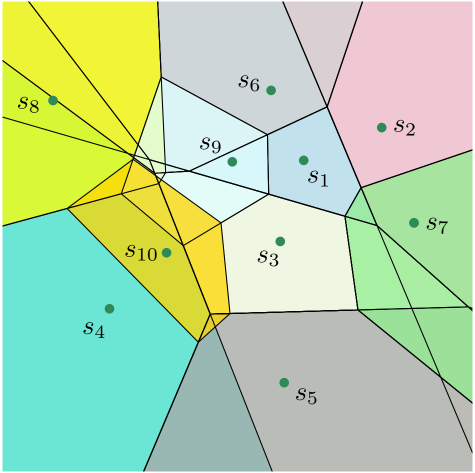


Definition (Candidate Set)

The ***candidate set*** for a weighted nearest neighbor of $q \in \mathbb{R}^2$ consists of all sites $s \in S$ such that all other sites in S either have a smaller weight or a larger Euclidean distance to q.

Candidate Sets



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- Only sites within the same candidate set may collide.

Lemma (Har-Peled and Raichel 2015)

For all points $q \in \mathbb{R}^2$, the candidate set for q among S is of size $\mathcal{O}(\log n)$ with high probability.

Overlay Arrangement

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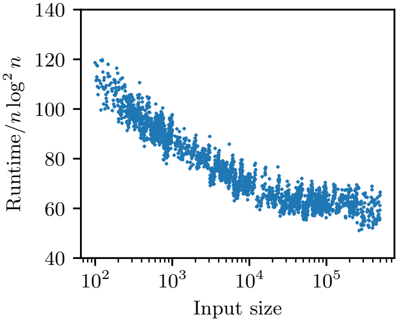
Theorem (Held and de Lorenzo 2020)

A wavefront-based approach allows to compute the MWVD of a set S of n (randomly) weighted point sites in expected $\mathcal{O}(n \log^4 n)$ time and expected $\mathcal{O}(n \log^3 n)$ space.

Experimental Evaluation

- The implementation is based on the Computational Geometry Algorithms Library (CGAL).
- We tested our strategy on over 3000 different inputs ranging from 256 vertices to 500 000 vertices.
- All tests were carried out on an Intel Core i9-7900X processor clocked at 3.3 GHz.
- For all of these inputs, the weights and point locations were chosen uniformly at random.



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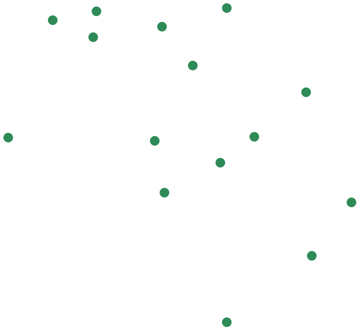

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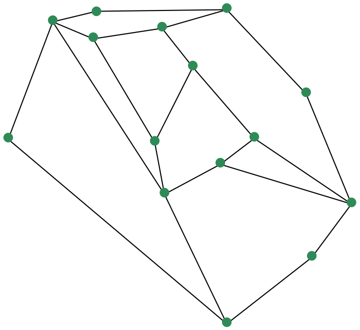
Given: A set S of n points in the plane.
Find: A plane graph with vertex set S (with each point in S having positive degree) that partitions the convex hull of S into the smallest possible number of convex faces. Note that collinear points are allowed on face boundaries, so all internal angles of a face are at most π.



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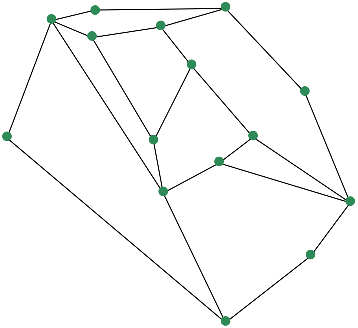


CG:SHOP 2020

Download | Submit Solution

Organized by: Erik Demaine (MIT), Sándor Fekete (TU Braunschweig), Phillip Keldenich (TU Braunschweig), Dominik Krupke (TU Braunschweig), Joseph S. B. Mitchell (Stony Brook University)

25 teams participating
Sept. 30, 2019, 6 p.m. (UTC) - Feb. 14, 2020, 11:59 a.m. (AoE)

view all competition news

The competition has ended. To view your score and the score of the best teams, please refer to the ranking tab.

Problem Description | Ranking | Instance Format

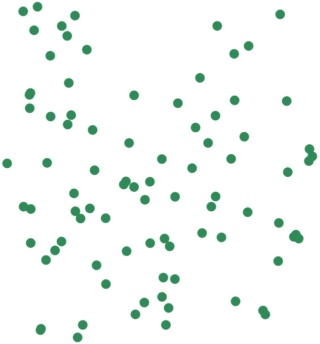
Minimum Convex Partition Problem

We are happy to announce the CG Challenge 2020, as part of CG Week in Zurich, Switzerland, June 22-26, 2020. As in the CG Challenge 2019, the objective will be to compute good solutions to instances of a difficult geometric optimization problem.

The contributors with the best solutions will be recognized at CG Week and invited to present their results. In addition, the top contributing teams will be invited to submit an extended abstract describing their methods and results, to be included in the LIPIcs proceedings of SoCG.

3-Approximation

- The 3APX tool implements the algorithm by Knauer and Spillner 2006.



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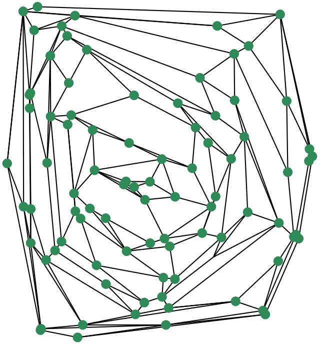
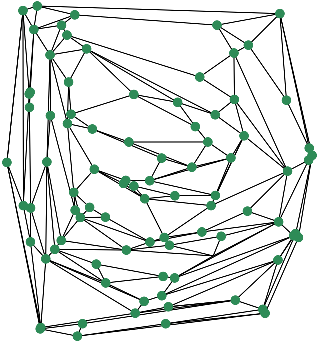


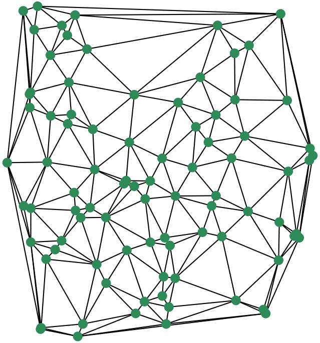
3-Approximation

- The 3APX tool implements the algorithm by Knauer and Spillner 2006.
- We extended 3APX by an approach based on onion layers.
- The decompositions generated contained lots of extremely long and thin triangles.



Merging Triangles

- ***Simple idea:*** Start with a Delaunay triangulation ...



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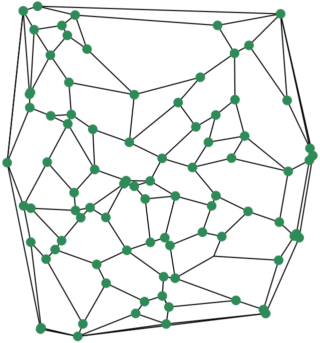
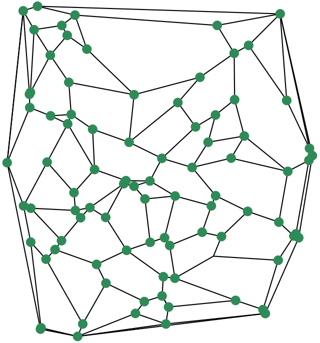


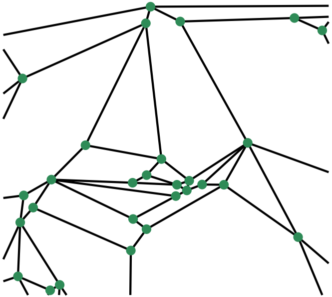
Merging Triangles

- ***Simple idea:*** Start with a Delaunay triangulation and merge neighboring faces.
- Our first implementation MERGEREFINE easily beat 3APX.
- This initial success motivated the development of a more sophisticated strategy.



An Improved Implementation

- RECURSOR introduces several heuristics.



- RECURSOR introduces several heuristics.
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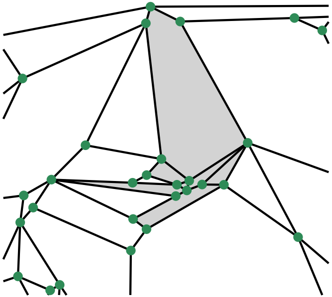

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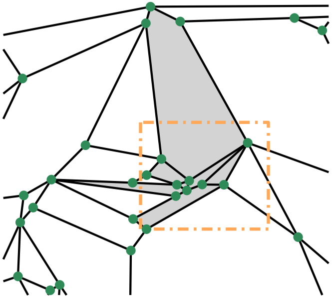

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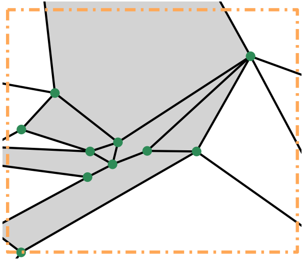

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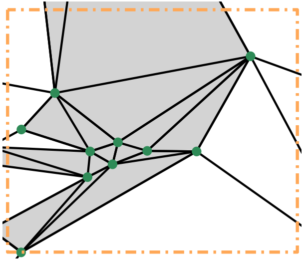

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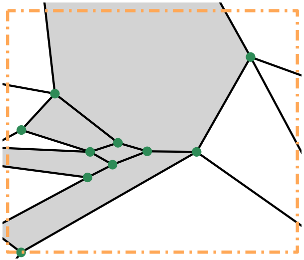

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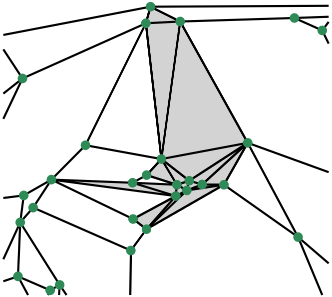

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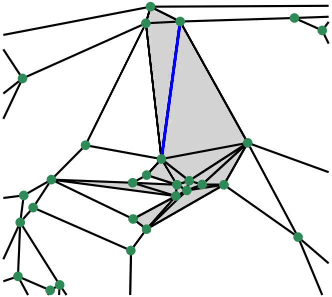

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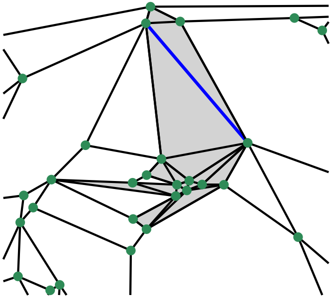

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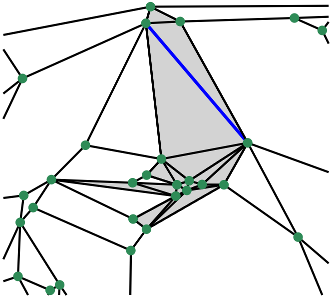

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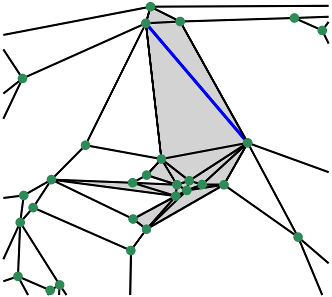
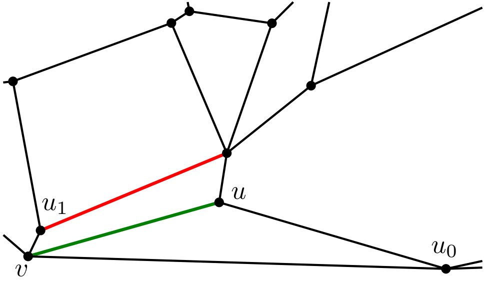
- ***Parallel recursor***: Partition a decomposition into several non-overlapping sets of faces.



Flipping Edges

- FLIPPER was implemented relatively late.
- It picks a high degree vertex and rotates incident edges.
- Unnecessary edges are removed.
- FLIPPER interacts with RECURSOR as it re-structures the respective decompositions.



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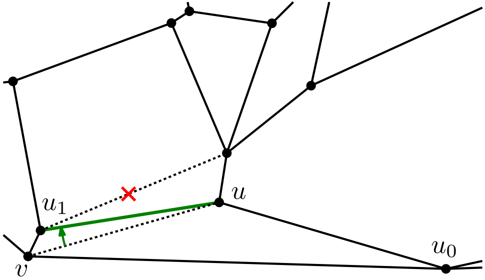
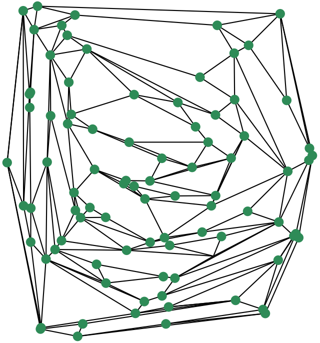


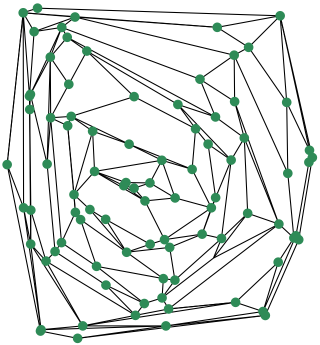
3APX (random)

#Faces 111



3APX (random) → 3APX (onion)

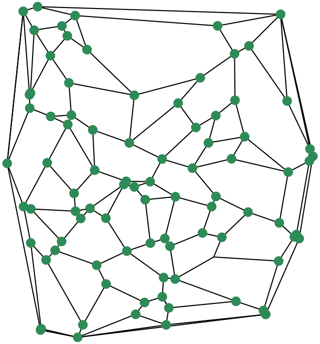
#Faces 100



Example Decomposition

3APX (random) → 3APX (onion) → MERGEREFINE

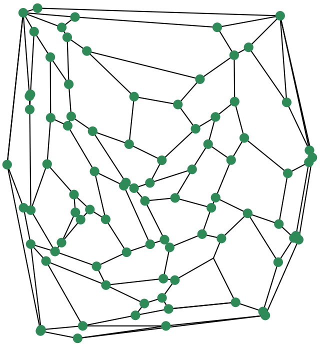
#Faces 63



Example Decomposition

3APX (random) → 3APX (onion) → MERGEREFINE → RECURSOR + FLIPPER

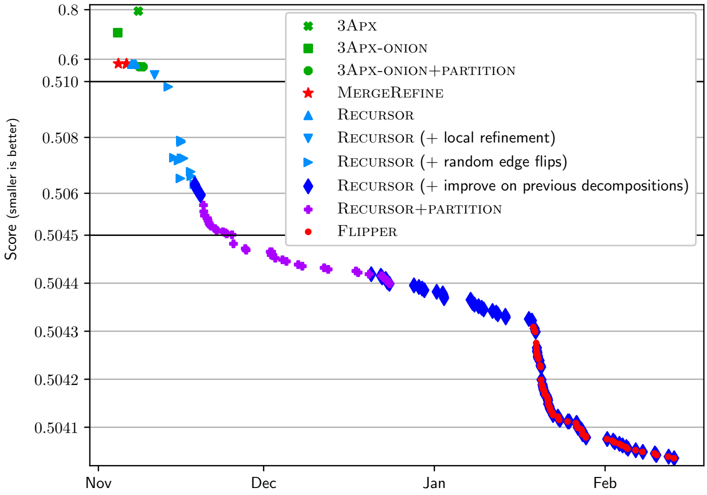
#Faces 54



Score Over Time

- We ran our tools on a wide variety of different computers.
- The estimated quality of a given decomposition is based on its ***score***.

$$\text{score} := \frac{\text{number of edges in convex partition}}{\text{number of edges in triangulation}}$$



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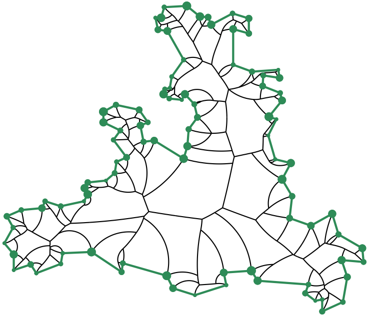

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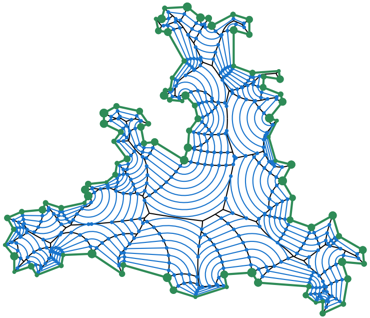
Variable-Radius Skeleton



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- It has a linear combinatorial complexity as its region stay connected in any case.

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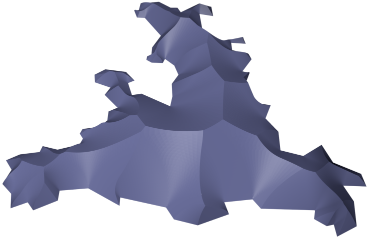

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- ***Variable-radius roofs*** can be derived from variable-radius skeletons.

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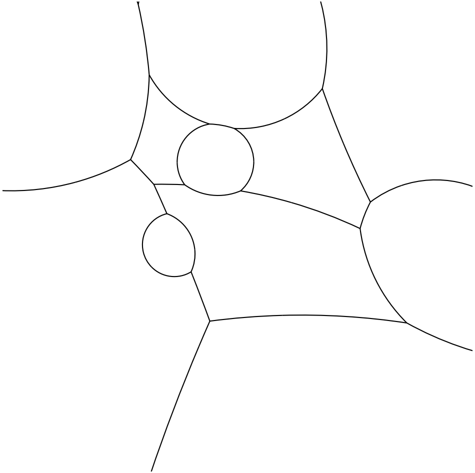
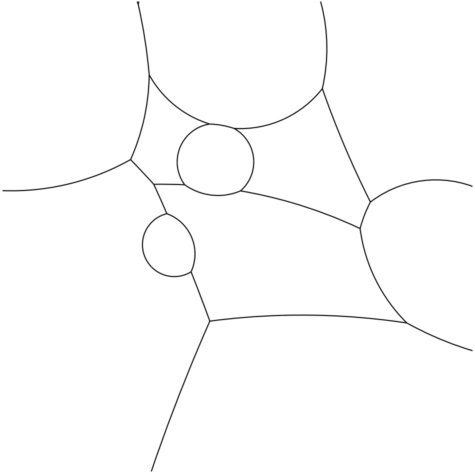
Published in *Proceedings of the 36th International Symposium on Computational Geometry (SoCG 2020)*



Definition (Weighted Bisector Graph)

A weighted bisector graph is a geometric graph whose faces are bounded by arcs that are portions of multiplicatively weighted bisectors of pairs of (point) sites such that each of its faces is defined by exactly one site.



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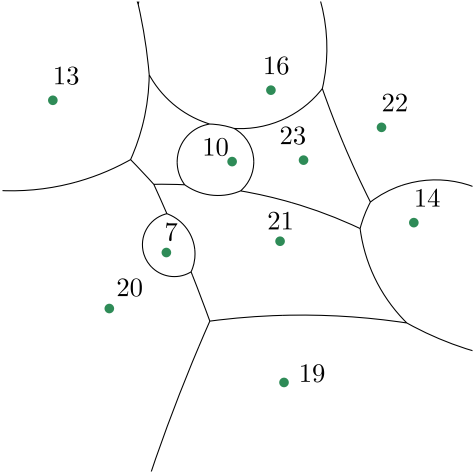
Problem

Given: A partition G of the plane into faces.



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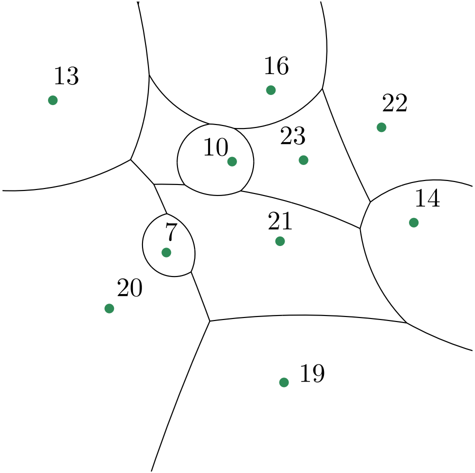
Given: A partition G of the plane into faces.
Find: A set of points and suitable weights such that G is a bisector graph of the weighted points, if a solution exists.



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Theorem (Eder, Held, de Lorenzo, and Palfrader 2021)

If G is a graph that is regular of degree three then we can decide in $\mathcal{O}(m)$ time whether it is a bisector graph, where m denotes the combinatorial complexity of G.

Overview

An Efficient, Practical Algorithm and Implementation for Computing Multiplicatively Weighted Voronoi Diagrams
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On the Recognition and Reconstruction of Weighted Voronoi Diagrams and Bisector Graphs
EDER, HELD, DE LORENZO, AND PALFRADER
Submitted to *Computational Geometry: Theory and Applications*

On the Generation of Spiral-Like Paths Within Planar Shapes
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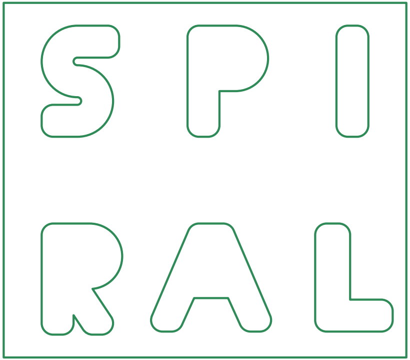

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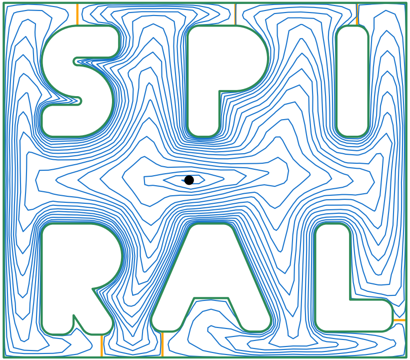
Given: A planar shape P that is bounded by straight-line segments and circular arcs as well as a ***step-over*** value $\delta > 0$.



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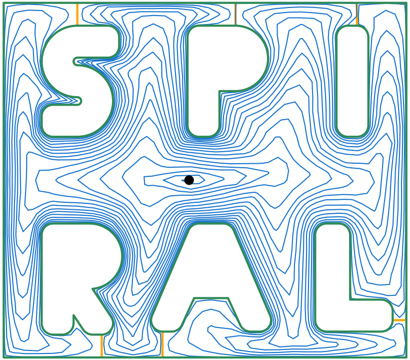
Find: A spiral path that (1) consists of straight-line segments, (2) has no self-intersections, (3) starts in the interior and ends at the boundary of the shape, and (4) respects the user-specified maximum step-over distance δ.



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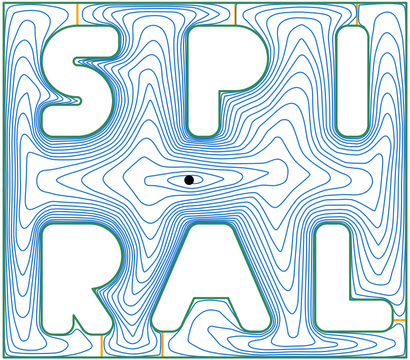
- Our approach is based on the medial axis within P.



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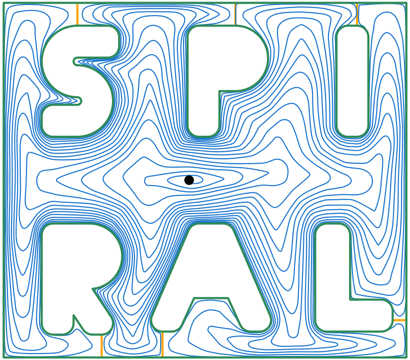
- Our approach is based on the medial axis within P.
- For ***high-speed machining***, it is important to avoid sharp corners.
- Thus, we smooth the spiral path using ***cubic B-splines***.



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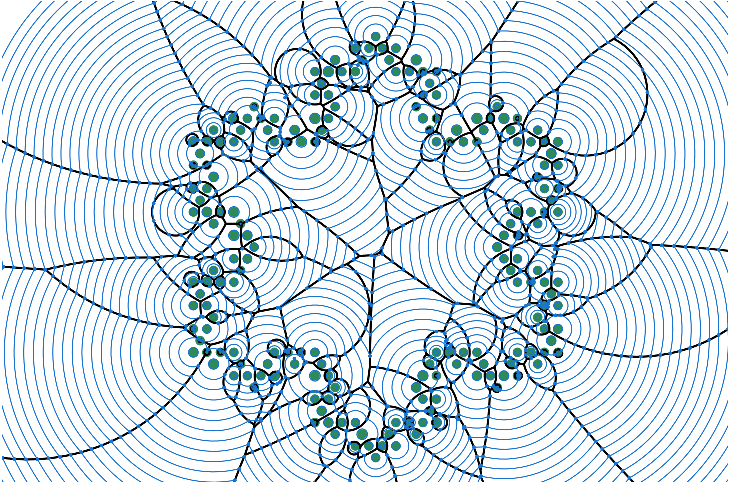
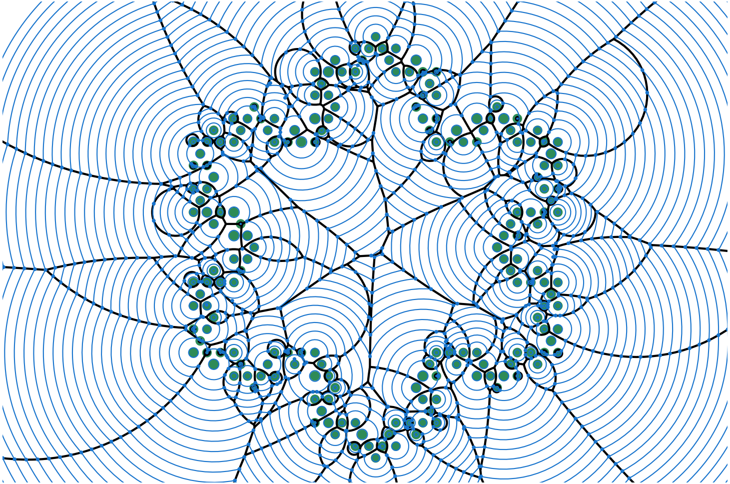
- Our approach is based on the medial axis within P.
- For ***high-speed machining***, it is important to avoid sharp corners.
- Thus, we smooth the spiral path using ***cubic B-splines***.
- Additionally, it is also possible to generate ***double spirals***.

The End





Thank you for your attention!

UNIVERSITÄT SALZBURG
Computational Geometry and Applications Lab

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